# Driver Steering Torque Estimation and Active FuzzyAssistance Control of Vehicle Path Tracking Systems

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**Abstract:** In this paper, an active fuzzy assistance method is proposed for vehicle path tracking system based on driver steering torque estimation. First of all, in order to describe the driver's steering behavior, vehicle kinematics and lateral dynamics, a driver-vehicle-road (DVR) system model is established. The parameter uncertainties are approximated by an interval type-2 (IT-2) fuzzy model. Secondly, to acquire the information of driver steering torque, an adaptive observer is introduced to estimate the driver steering torque effectively. At the same time, the unmeasured system states are also estimated by the observer. Then, an adaptive steering controller is proposed to reduce the driver's burden and enhance the performance of vehicle path tracking. Finally, the effectiveness of the proposed method is verified by simulation.

Keywords: vehicle dynamics control, path tracking control, assistance control, torque estimation, fuzzy model.

# 1 Introduction

In order to ensure the driving safety, drivers must maintain control of the vehicle before the advent of the era of fully automatic driving. However, under the influence of various factors, drivers may still make operational errors [1, 2]. To reduce the driver's driving burden and reduce the occurrence of traffic accidents, various advanced driver assistance systems have been developed [3]. Therefore, the driver assistance system has been paid more and more attention by the academia and the industry.

The path tracking is one of the basic functions of autonomous vehicles <sup>[4]</sup>. In such as a scenario, the driver assistance control objective is to assist the driver to keep the path tracking error as close to zero as possible by adjusting the heading and lateral position of the vehicle <sup>[5, 6]</sup>. In order to design the driver assistance controller, it is necessary to establish an accurate DVR system model. Li et al. <sup>[7]</sup> employed a Takagi-Sugeno (T-S) fuzzy model to characterize the uncertain dynamics of the DVR system by considering both uncertainties in parameters and sampled measurements. The T-S fuzzy model can describe the nonlinearity and uncertainty of the system. And a T-S fuzzy model was constructed for the vehicle path tracking system with communication delay in <sup>[8]</sup>. Nguyen et al. <sup>[9]</sup> studied lateral shared control between human drivers and intelligent vehicle lane keeping and obstacle avoidance. The T-S fuzzy control method was proposed for the change of driver activity parameters and vehicle speed with time. However, due to the limitation of sensor performance, it is difficult to measure vehicle longitudinal speed accurately. The inaccurate longitudinal velocity may cause some uncertainties in the membership function of the above T-S fuzzy vehicle model. The IT-2 fuzzy model can effectively deal with the uncertainty of membership functions and is investigated in this work.

The driver's steering torque is widely used to evaluate the driver's attention. To identify the human steering intervention torque, the authors in [10] proposed a model-based estimation strategy for the artificial steering intervention

torque of autonomous vehicles. In order to improve the electric power assisted steering system performance and reduce system complexity, the authors in [11] proposed a sliding mode observer to estimate the driver torque and road reaction of electric power steering systems with consideration of unknown inputs. In [12], the authors designed an unknown input observer using a decoupling-based technique to estimate the drive torque and sideslip angle. The above results can accurately estimate the steering torque of the driver and play a great role in explaining the driver's attention. Based on the adaptive torque estimation method, the observer can provide real-time dynamic information of the driver's torque by designing the adaptive updating law of the driver's torque [13]. On this basis, this paper studies an estimation method of driver's steering torque, which is of great significance to determine the active input required by the auxiliary driver. Based on this, this paper proposes an active fuzzy assistance control method based on driver's steering torque observer, which plays an important role in require driver's steering torque and improving vehicle path tracking performance.

The rest of this paper is organized as follows. In Section 2, establish the driver-vehicle-road interval type-2 fuzzy model. Section 3 gives the design results of the observer and controller. Section 4 introduces the simulation results of the designed control method, followed by some concluding remarks in Section 5.

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	Table 1. Nomenclature	
Notation	Definition	Unit
m	Vehicle mass	kg
$v_y$	Vehicle lateral velocity	m/s
$v_x$	Vehicle longitudinal velocity	m/s
$\psi_v$	vehicle heading angle	rad
$I_z$	Yaw moment inertia	$kg\!\cdot\! m^2$
$y_e$	Lateral deviation error at $l_d$	m
$\psi_{e}$	Heading error at current position	rad
$l_d$	look-ahead distance	m
$C_a$	Cornering stiffness of front wheel	N/rad
$C_{\scriptscriptstyle b}$	Cornering stiffness of rear wheel	N/rad
$\delta_{\scriptscriptstyle d}$	front wheel steering angle	rad

# 2 Problem formulation

### 2.1 Driver-vehicle-road System model

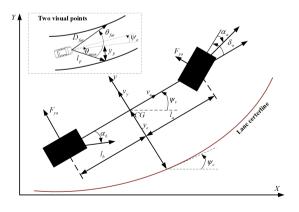


Figure 1. Vehicle lateral kinematics and dynamics model

The single-track model shown in Fig.1 can explain the lateral behavior of the vehicle. It is assumed that both the driver steering angle and sideslip angle are small, and the aerodynamics and vehicle longitudinal dynamics are ignored. The lateral dynamics and kinematics of the vehicle can be expressed as:

$$\begin{cases}
m(\dot{v}_y + v_x \dot{\psi}) = C_f \left(\delta_d - \frac{l_f \dot{\psi} - v_y}{v_x}\right) + C_r \left(\frac{l_r \dot{\psi} - v_y}{v_x}\right) \\
I_z \dot{\psi} = l_f C_f \left(\delta_d - \frac{l_f \dot{\psi} - v_y}{v_x}\right) - l_r C_r \left(\frac{l_r \dot{\psi} - v_y}{v_x}\right)
\end{cases} \tag{1}$$

$$\begin{cases} \dot{\psi}_e = \dot{\psi}_v - v_x \rho(\sigma) \\ \dot{y}_e = v_v + v_x \psi_e + l_d \dot{\psi} \end{cases}$$
 (2)

The steering system is described as follows:

$$I_{s}\ddot{\delta}_{w} + B_{s}\dot{\delta}_{d} = \frac{T_{d}(t) + T_{a}(t)}{R_{s}} - \frac{C_{a}\eta_{t}}{R_{s}^{2}} \left(\delta_{w} - \frac{v_{y}}{v_{x}} - \frac{l_{a}\dot{\psi}_{v}}{v_{x}}\right)$$
(3)

where  $I_s$  is the moment of inertia,  $B_s$  is the damping coefficient of the steering system, and  $R_s$  is the steering transmission ratio.

The driver torque is described as:

$$T_{d}(t) = G_{c} + G_{a} = k_{s1}\theta_{near} + k_{s2}\theta_{far}$$
(4)

where  $G_c$  and  $G_a$  as the compensatory and anticipatory components of the driver's behavior, respectively.  $k_{s1} = -300$ ,  $k_{s2} = -2$  are the driver's proportional actions in accordance with  $\theta_{near}$  and  $\theta_{far}$ , separately. To replicate the steering actions of drivers, a two-point preview model is employed, and the compensation behavior and expected behavior are described by near point and far point respectively. Near and far point is defined as:

$$\begin{cases} \theta_{near} = \psi_e + \frac{y_p}{l_p} \\ \theta_{for} = \theta_1 v_y + \theta_2 \dot{\psi}_y + \theta_3 \delta_d \end{cases}$$
 (5)

where  $\theta_1 = \tau_a^2 a_{21}$ ,  $\theta_2 = \tau_a + \tau_a^2 a_{22}$ ,  $\theta_3 = \tau_a^2 a_{25}$  and  $\tau_a = 0.5$ , representing the anticipatory time.

In summary, the human-vehicle road model can be derived as follows:

$$\dot{x}_{p}(t) = A_{p}x_{p}(t) + B_{p1}\omega_{s}(t) + B_{p2}(T_{d}(t) + T_{a}(t))$$
(6)

where the system state vector and matrices are:

$$a_{11} = -\frac{C_a + C_b}{mv_x}, a_{12} = \frac{l_b C_b - l_a C_a}{mv_x} - v_x, a_{15} = \frac{C_a}{m}$$

where  $a_{22} = \frac{l_b C_b - l_a C_a}{I_s v_s}$ ,  $a_{23} = -\frac{l_a^2 C_a + l_b^2 C_b}{I_s v_s}$ ,  $a_{25} = \frac{l_a C_a}{I_s}$ ,  $a_{66} = -\frac{B_s}{I_s}$ 

$$a_{61} = \frac{C_a \eta(t)}{I_s R_s^2 v_x}, a_{62} = \frac{C_a \eta(t) I_a}{I_s R_s^2 v_x}, a_{65} = -\frac{C_a \eta(t)}{I_s R_s^2}, b_{61} = \frac{1}{I_s R_s}$$

The control output defined as follow:

$$y_p(t) = C_p x_p(t) \tag{7}$$

where  $C_p = [0_{5\times 1} \ I_{5\times 5}]$ .

# 2.2 IT-2 fuzzy model

Considering the tire cornering stiffness varies along with the road friction,  $C_a$  and  $C_b$  are redescribed to establish an efficient T-S fuzzy model as [14]:

$$C_a = \sigma_1(t)C_{ao}, C_b = \sigma_1(t)C_{bo} \tag{8}$$

Three prerequisite variables,  $\sigma_1(t)$ ,  $\sigma_2(t) = v_x$  and  $\sigma_3(t) = \sigma_1(t)/v_x$ , are selected to describe the model parameters generated by the time-varying parameters  $v_x$ ,  $C_a$  and  $C_b$ . In practice,  $\sigma(t)$  is difficult to measure, and due to sensor limitations, longitudinal velocity is difficult to measure accurately, resulting in uncertainty of membership function. Therefore, the IT-2 fuzzy model is used to deal with uncertainty. The three premise variables are represented as:

$$\sigma_i(t) = \theta_{is}(t)\sigma_i + \theta_{ib}(t)\sigma_i \tag{9}$$

where i = 1, 2, 3, the maximum and minimum values of  $\sigma_i(t)$  are  $\sigma_i$  and  $\sigma_i$ , the corresponding membership functions are represented by  $\theta_{is}(t)$  and  $\theta_{ib}(t)$ . The membership function is calculated using the following formula:

$$\theta_{is}(t) = \frac{\breve{\sigma}_i - \sigma_i(t)}{\breve{\sigma}_i - \sigma_i}, \theta_{ib}(t) = \frac{\sigma_i(t) - \sigma_i}{\breve{\sigma}_i - \sigma_i}$$
(10)

Regarding  $\sigma_1(t)$  and  $\sigma_2(t)$ , the lower and upper membership functions are presented as follows (i = 1, 2):

$$\theta_{is}^{L}(t) = \theta_{is}(t)\Big|_{+\Delta}, \theta_{is}^{U}(t) = \theta_{is}(t)\Big|_{-\Delta} 
\theta_{ib}^{L}(t) = \theta_{ib}(t)\Big|_{-\Delta}, \theta_{ib}^{U}(t) = \theta_{ib}(t)\Big|_{-\Delta}$$
(11)

The IT-2 fuzzy model as follows:

Model Rule i: IF  $\sigma_1(t)$  is "Small or Big",  $\sigma_2(t)$  is "Small or Big" and  $\sigma_3(t)$  is "Small or Big", THEN  $\dot{x}_p(t) = A_p x_p(t) + B_{p1} \omega_s(t) + B_{p2} (T_d(t) + T_a(t)) \tag{12}$ 

where i = 1, 2, ..., 8,  $A_{pli}$ ,  $B_{pli}$  and  $B_{p2i}$  are obtained by replacing  $\sigma_1(t)$ ,  $\sigma_2(t)$  and  $\sigma_3(t)$  with corresponding values in matrixes right after (5), respectively. For the i-th model rule, the firing strength is

$$\hat{\phi}(t) = \left[\prod_{i=1}^{3} \theta_{i}^{L}(t), \prod_{i=1}^{3} \theta_{i}^{U}(t)\right] = \left[\phi^{L}(t), \phi^{U}(t)\right]$$
(13)

The overall IT-2 fuzzy model is shown as:

$$\dot{x}_{p}(t) = \sum_{i=1}^{8} \phi_{i}(t) [A_{pi} x_{p}(t) + B_{p1i} \omega_{s}(t) + B_{p2i} (T_{a}(t) + T_{d}(t))]$$
(14)

 $\text{where} \quad \phi_i = \frac{\varsigma_i(t)\phi_i^L(t) + (1-\varsigma_i(t))\phi_i^U(t)}{\sum_{k=1}^r \left(\varsigma_k(t)\phi_k^L(t) + (1-\varsigma_k(t))\phi_k^U(t)\right)}, \text{ where} \quad \varsigma_i(t) \in [0,1] \quad \text{is a nonlinear function that depends on parametric } \int_{k=1}^r \left(\varsigma_k(t)\phi_k^L(t) + (1-\varsigma_k(t))\phi_k^U(t)\right),$ 

uncertainty.  $\phi_i(t)$  is simplified as  $\phi_i$  and satisfies  $0 \le \xi \le 1$ .

# 2.3 Driver steering torque estimation observer

The observer is constructed as follows:

$$\begin{cases}
\dot{\hat{x}}_{p}(t) = \sum_{i=1}^{8} \sum_{j=1}^{8} \zeta_{i} \zeta_{j} \{A_{pi} x_{p}(t) + B_{p2i} [\hat{T}_{d}(t) + T_{a}(t)] \\
+ R_{j} [y_{p}(t_{k}h) - \hat{y}_{p}(t - \tau(t))] \} \\
\dot{\hat{T}}_{d}(t) = \Gamma \sum_{j=1}^{8} \zeta_{j} F_{j} [(y_{p}(t_{k}h) - \hat{y}_{p}(t - \tau(t))) \\
\hat{y}_{p}(t) = C_{p} \hat{x}_{p}(t) \\
e_{y}(t) = y_{p}(t_{k}h) - y_{p}(k - \tau(t))
\end{cases} \tag{15}$$

By defining the estimated error as:  $w(t) = \left[ (x_p(t) - \hat{x}_p(t))^T \quad (T_d(t) - \hat{T}_d(t))^T \right]^T$ , and  $g(t) = \left[ \omega_s(t)^T \quad \dot{T}_d^T(t) \right]^T$ , the augmented error dynamic system is obtained as:

$$\dot{w}(t) = \sum_{i=1}^{8} \sum_{j=1}^{8} \zeta_i \zeta_j \{ A_w w(t) + B_w w(t - \tau(t)) + Gg(t) + F_w e_y(t) \}$$
(16)

where  $A_{w} = \begin{bmatrix} A_{p} & B_{p2} \\ 0 & 0 \end{bmatrix}, B_{w} = -\tilde{R}_{j}C_{p}, G = \begin{bmatrix} B_{p1} & 0 \\ 0 & I \end{bmatrix}, F_{w} = -\tilde{R}_{j}, \tilde{R}_{j} = \begin{bmatrix} R_{j} \\ F_{j} \end{bmatrix}$ 

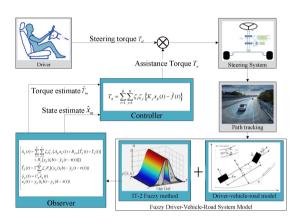


Figure 2. Observer-based fuzzy assistance steering control structure

# 3 Main result

# 3.1 Adaptive observer design

In this section, an asymptotically stable IT-2 fuzzy driver steering estimation observer is designed, which satisfies the following constraints:

Theorem 1: Given positive scalars  $\gamma_1$ ,  $\gamma_2$ ,  $\varepsilon_m$ ,  $\varepsilon_u$ ,  $\varepsilon_M$ , if there exist positive matrices P,  $G_1$ ,  $G_2$ ,  $G_3$ ,  $Q_1$ ,  $Q_2$ ,  $Q_3$  and general matrices J, Y, H such that

$$\Xi_{\perp} < 0 \tag{18}$$

$$\Psi_{r} < 0 \tag{19}$$

$$\Upsilon_{r} < 0 \tag{20}$$

$$\begin{bmatrix} G_{1} & H \\ * & G_{2} \end{bmatrix} > 0$$

$$\Xi_{ij} = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & \Xi_{23} \\ * & * & \Xi_{33} \end{bmatrix}, \Xi_{11} = \begin{bmatrix} \Xi & P - Y + A_{s}^{T}Y^{T} \\ * & \varepsilon_{m}^{2}G_{1} + \varepsilon_{u}^{2}G_{2} + \varepsilon_{m}^{2}G_{3} - [Y]_{s} \end{bmatrix}, \Xi_{12} = \begin{bmatrix} Q_{1} & \frac{\pi^{2}}{4}Q_{3} - JC \\ * & -JC \end{bmatrix}, \Xi_{13} = \begin{bmatrix} 0 & VL_{s} \\ 0 & VL_{s} \end{bmatrix}$$

$$\Xi_{22} = \begin{bmatrix} Q_{2} - Q_{1} - G_{1} - G_{2} & G_{2} - H \\ * & [H - Q_{2}]_{s} - \frac{\pi^{2}}{4}Q_{3} \end{bmatrix}, \Xi_{23} = \begin{bmatrix} H & 0 \\ Q_{2} - H & 0 \end{bmatrix}, \Xi_{33} = diag\{-Q - Q_{2}, -\gamma^{2}I\}\}$$

$$\Xi = Q_{1} - G_{1} - \frac{\pi^{2}}{4}G_{3} + [YA_{s}]_{s} + E_{1}^{T}E_{1}, \Psi_{ij} = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} \\ * & \Psi_{22} & \Psi_{23} \\ * & * & \Psi_{33} \end{bmatrix}, \Psi_{11} = \begin{bmatrix} \Psi & P - Y + A_{s}^{T}Y^{T} \\ * & \varepsilon_{m}^{2}G_{1} + \varepsilon_{u}^{2}G_{2} + \varepsilon_{M}^{2}G_{3} - [Y]_{s} \end{bmatrix}$$

$$\Psi = Q_{1} - G_{1} - \frac{\pi^{2}}{4}G_{3} + [YA_{s}]_{s} + E_{2}^{T}E_{2}, \Psi_{12} = \Xi_{12}, \Psi_{22} = \Xi_{22}, \Psi_{23} = \Xi_{23}, \Psi_{33} = diag\{-Q_{2} - G_{2}, -\gamma^{2}I\}, E_{1} = \begin{bmatrix} 0 & I \end{bmatrix}, E_{2} = \begin{bmatrix} I & 0 \end{bmatrix}, Y_{11} = \Psi_{11}, Y_{12} = \Psi_{12}, Y_{13} = \Psi_{13}, Y_{22} = \Psi_{22}, Y_{23} = \Psi_{23}$$

Moreover, the observer gain  $R_i$  can be derived from  $R_i = Y^{-T}J$ .

**Proof.** Consider building a positive Lyapunov function V(t).

$$\begin{split} V(t) &= \boldsymbol{w}^T(t) P \boldsymbol{w}(t) + \int_{t-\varepsilon_m}^t \boldsymbol{w}^T(s) Q_1 \boldsymbol{w}(v) ds + \int_{t-\varepsilon_m}^{t-\varepsilon_m} \boldsymbol{w}^T(s) Q_2 \boldsymbol{w}(v) dv \\ &+ \varepsilon_m \int_{-\varepsilon_m}^0 \int_{t+\theta}^t \dot{\boldsymbol{w}}^T(v) G_1 \dot{\boldsymbol{w}}^T(v) dv d\theta + \varepsilon_M \int_{-\varepsilon_u}^{-\varepsilon_m} \int_{t+\theta}^t \dot{\boldsymbol{w}}^T(v) G_2 \dot{\boldsymbol{w}}(v) dv d\theta \\ &- \frac{\pi^2}{4} \int_{nh}^t [\boldsymbol{w}(v) - \boldsymbol{w}(nh)]^T G_3 [\boldsymbol{w}(v) - \boldsymbol{w}(nh)] dv + \varepsilon_u^2 \int_{nh}^t \dot{\boldsymbol{w}}^T(v) G_3 \dot{\boldsymbol{w}}(v) dv dv dv \end{split}$$

Time derivative of V(t) is obtained as

$$\dot{V}(t) = 2w^{T}(t)P\dot{w}(t) + w^{T}(t)Q_{1}w(t) - w^{T}(t - \varepsilon_{u})Q_{2}w(t - \varepsilon_{u}) + w^{T}(t - \varepsilon_{m})(Q_{2} - Q_{1})w(t - \varepsilon_{m})$$

$$-\varepsilon_{m}\int_{t-\varepsilon_{m}}^{t}\dot{w}^{T}(v)G_{1}\dot{w}(v)dv - \varepsilon_{M}\int_{t-\varepsilon_{u}}^{t-\varepsilon_{m}}\dot{w}^{T}(v)G_{2}\dot{w}(v)dv + \varepsilon_{m}^{2}\dot{w}^{T}(v)G_{1}\dot{w}(v) + \varepsilon_{M}^{2}\dot{w}^{T}(v)G_{2}\dot{w}(v)$$

$$+d_{u}^{2}\dot{w}^{T}(v)G_{3}\dot{w}(v) - \frac{\pi^{2}}{4}[w(t) - w(t - d(t))]^{T}G_{3}[w(t) - w(t - d(t))]$$
(22)

For the general matrix Y we can get the following equation:

$$[w^{T}(t)Y + \dot{w}^{T}(t)\lambda Y][A_{w}w(t) + B_{w}w(t - \tau(t)) + Gg(t) + F_{w}e_{v}(t) - \dot{w}(t)] = 0$$
(23)

By combining the above formula, by defining  $J = Y^T \tilde{L}$  and  $e_v(t) = 0$ , we can get:

$$\dot{V}(t) + [T_d(t) - \hat{T}_d(t)]^T [T_d(t) - \hat{T}_d(t)] - \gamma_1^2 g^T(t) g(t) \le \sum_{i=1}^q \sum_{j=1}^q \zeta_i \zeta_j \varphi_1^T(t) \Xi_r \varphi_1(t)$$
(24)

where  $\varphi_1^T(t) = col\{w(t), \dot{w}(t), w(t-d_m), w(t-d(t)), w(t-d_u), \omega_s(t)\}$ . Under zero initial conditions and  $\Xi_r < 0$  in (18), it can effectively guarantee the performance of  $\|T_d(t) - \hat{T}_d(t)\|_2 \le \gamma_1 \|e_y(t)\|_2$  in (17).

Secondly, consider g(t) = 0 in the constraint (17), the following function  $J_2(t)$  satisfies

$$\dot{V}(t) + [T_d(t) - \hat{T}_d(t)]^T [T_d(t) - \hat{T}_d(t)] - \gamma_2^2 \omega_s^T(t) \omega_s(t) \le \sum_{i=1}^q \sum_{j=1}^q \zeta_i \zeta_j \varphi_2^T(t) \Psi_r \varphi_2(t)$$
(25)

where  $\varphi_2^T(t) = col\{w(t), \dot{w}(t), w(t-d_m), w(t-d(t)), w(t-d_u), e_y(t)\}$ . Under  $\Psi_r < 0$  in (19), it allows the constraint  $\|T_d(t) - \hat{T}_d(t)\|_2 \le \gamma_2 \|g(t)\|_2$  in (17) can be guaranteed.

Similarly, when g(t) = 0, the following condition is true as:

$$\dot{V}(t) + [x_p(t) - \hat{x}_p(t)]^T [x_p(t) - \hat{x}_p(t)] - \gamma_3^2 \omega_s^T(t) \omega_s(t) \le \sum_{i=1}^q \sum_{j=1}^q \zeta_i \zeta_j \varphi_3^T(t) \Upsilon_r \varphi_3(t)$$
(26)

Therefore, the above proof can ensure that the error dynamics in (16) is stable and the constrains in (17) are guarantee. Proof complete.

# 3.2 Adaptive controller design

Substituting (4) into (6), the driver-vehicle model is obtained as [15]:

$$\dot{x}_{n}(t) = \bar{A}_{n} x_{n}(t) + B_{n1} \omega_{s}(t) + B_{n2} (T_{n}(t) + \tilde{f}(t))$$
(27)

where

$$\vec{A}_p = \begin{bmatrix}
a_{11} & a_{12} & 0 & 0 & a_{15} & 0 \\
a_{22} & a_{23} & 0 & 0 & a_{25} & 0 \\
1 & l_d & 0 & v_x & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 \\
\hat{a}_{61} & \hat{a}_{62} & \lambda k_{s1} & \lambda T_{s3} & \hat{a}_{65} & a_{66}
\end{bmatrix}$$

$$\hat{a}_{61} = a_{61} + \lambda T_{s1}, \hat{a}_{62} = a_{62} + \lambda T_{s2}, \hat{a}_{65} = a_{65} + \lambda T_{s4}, \hat{T}_d(t) - T_d(t) = \tilde{f}(t)$$

By estimating the steering torque of the driver, an adaptive controller can be designed. Firstly, the proposed controller is as follows:

$$T_{a} = \sum_{i=1}^{8} \sum_{j=1}^{8} \zeta_{i} \zeta_{j} \left\{ K_{j} x_{p}(t) - \tilde{f}(t) \right\}$$
 (28)

where the local control gain  $K_i$  needs to be designed.

Therefore, combining equations (26) and (27), the IT-2 fuzzy model of the closed-loop system can be described as:

$$\dot{x}_{p}(t) = \sum_{i=1}^{8} \sum_{j=1}^{8} \zeta_{i} \zeta_{j} \{ (\overline{A}_{p} + B_{p2} K_{j}) x_{p}(t) + B_{p1} \omega_{s}(t) \}$$
(29)

In brief, the adaptive steering controller is proposed to make the system (26) be stable and satisfy the following performance as:

$$\left\| y_p(t) \right\|_2 < \gamma \left\| \omega_s(t) \right\|_2 \tag{30}$$

Theorem 2: Given positive scalars  $\gamma^3, \alpha_1, \alpha_2$ , if there exist positive matrices  $\overline{W}_1, \overline{W}_2, \overline{W}_3, \overline{N}_1, \overline{N}_2$  and general matrices  $\overline{U}, \overline{G}$  such that (i, j = 1, 2, ..., 8)

$$\overline{\Phi}_{\cdot \cdot} < 0 \tag{31}$$

$$\overline{\Phi}_{ij} + \overline{\Phi}_{ji} < 0, i < j \tag{32}$$

$$\bar{\Phi}_{ij} = \begin{bmatrix} \bar{\Theta}_{ij}^{11} & \bar{\Theta}_{ij}^{12} & \bar{\Theta}_{ij}^{13} \\ * & \bar{\Theta}_{22} & 0 \\ * & * & \bar{\Theta}_{33} \end{bmatrix}$$
(33)

where

$$\overline{\Theta} = [\overline{R}(\overline{A}_{pi} + B_{2i}\overline{K}_{j})]_{s} + \overline{N}_{1} - \overline{W}_{1} - \frac{\pi^{2}}{4}\overline{W}_{3} + C^{T}C, \overline{\Theta}_{22} = diag\{-\overline{W}_{1}^{-1}, -\overline{W}_{2}^{-1}, -\overline{W}_{3}^{-1}\}, \overline{\Theta}_{33} = -I$$

Then, the system in (28) satisfies the asymptotic stability and the constrain  $\|y_p(t)\|_2 < \gamma \|\omega_s(t)\|_2$ . Moreover, the feedback gain  $K_j$  of the controller can be estimated as  $K_j = \overline{K}_j \overline{R}^{-1}$ .

#### 4 Performance simulation result

In this part, the effectiveness of the proposed assistance control method is proved by simulation. Table 2 gives the model parameters used in the simulation.

Fig. 3 (a) provides the variation result of road curvature in a lane change maneuver. Fig. 3 (b) and Fig. 3 (c) depicts the simulation results of the driver's steering torque and assistance torque. It can be seen from Fig. 3 (b) that the steering torque with control is less than without control. Therefore, the proposed control method can reduce the driver's torque and thus improve the driving comfort.

Fig. 4 shows the path tracking performances such as (a) lateral velocity, (b) heading error, (c) lateral offset, (d) yaw rate, (e) front wheel steering rate, and (f) front wheel steering angle. As can be seen from Fig. 4(a) and (b), the proposed control method can effectively reduce the lateral offset and lateral velocity, and improve the path tracking effect. From the Fig. 4(b) and (d), the adaptive controller can reduce the yaw rate and heading error, and make their changes smoother, indicating that the controller has good stability. Fig. 4(e) and (f) shows front wheel steering rate and front wheel steering angle, respectively. This indicates that the front wheel steering angle falls within the acceptable range.

The global trajectory of the vehicle is shown in Figure 5, which shows that the driver can use adaptive control methods to achieve the desired path. However, the proposed adaptive control method produces less fluctuation than the uncontrolled method. This also verifies the superiority of the adaptive control method proposed in this paper in improving the performance of path tracking.

Table 1. Model parameters

Symbol	value	Symbol	value	
	1370 kg	$I_s$	2315 kg·m <sup>2</sup>	
$l_{\scriptscriptstyle m}$	1.4 m	$l_n$	1.5 m	
$C_{\scriptscriptstyle m}$	67208 N/rad	$C_{n}$	66218 N/rad	
$l_{\scriptscriptstyle d}$	5 m	$I_s$	$0.05~kg\!\cdot\!m^2$	
$B_{s}$	5.73 Nm/rad/s	$R_s$	22	
$oldsymbol{\eta}_{\scriptscriptstyle t}$	0.185 m	$T_{_{I}}$	3 s	
$T_{i}$	0.3 s	$T_{n}$	0.1 s	
$K_c$	10	$K_{\scriptscriptstyle a}$	30	
	0.6 0.4 0.2 0.2 0.2 0.2 0.2		Assistance torque (m.N)anibu o o o o o o o o o o o o o o o o o o o	Adaptiv

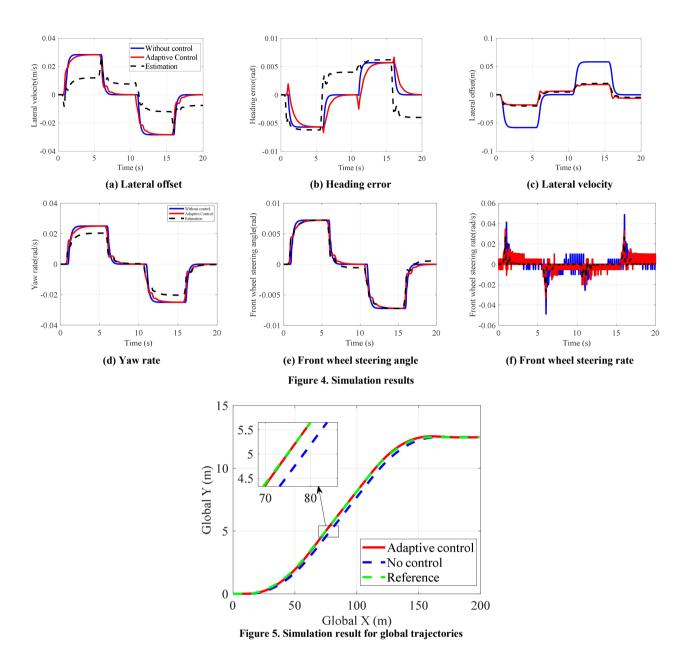
(b) Steering torque Figure 3. Simulation inputs for road curvature and simulation results

Time (s)

(c) Assistance torque

Time (s)

(a) Road curvature



# **5** Conclusion

This paper presented an observe-based driver adaptive assistance control method for path tracking system. Firstly, considering the driver's steering behavior, vehicle kinematics and lateral dynamics, an interval Type-2 fuzzy model was established to describe the global driver-vehicle-road system model, which can effectively deal with the system uncertainty caused by the time-varying characteristics of longitudinal velocity. Secondly in order to obtain the driver's steering torque information, an adaptive observer was designed to effectively estimate the driver's steering torque. The obtained observer has a great contribution to improving the accuracy of the system state and driver's steering torque estimation. Then, by solving linear matrix inequalities, an adaptive steering control method was proposed to reduce the driver's burden and improve the performance of vehicle path tracking. Finally, the effectiveness of the proposed control method was verified by simulation.

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