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Improved Design of Generalized Dynamic Rollover Threshold of Multi-Axial Vehicle

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Abstract: This paper aims at proposing an improved generalized dynamic rollover threshold of multi-axial vehicles based on lateral load transfer ratio (LTR) and providing the foundation for rollover prediction. The study is carried out by utilizing vehicle handling dynamics to build a vehicle rollover model, which takes into account the characteristics of suspension and limit equilibrium of tires' lifting-off. We develop a real-time rollover warning platform to dynamically indicate the vehicle rollover trend and compare our generalized threshold with two other common used thresholds, not only on TruckSim platform with simulation vehicle, but also on the rollover warning platform with test vehicle. Results show that on TruckSim platform, the proposed LTR is much closer to the defined LTR, especially when facing rollover, it's much more sensitive and accurate in indicating impending danger of rollover; on the rollover warning platform, three LTRs have similar trends, while the proposed LTR performs better in predicting rollover.

Keywords: multi-axle vehicle; dynamic rollover threshold; lateral load transfer ratio (LTR); Rollover warning platform (RWP)

1 Introduction

Rollovers are dangerous incidents and have a higher fatality rate than other kinds of crashes. Vehicles with high center of gravity are more prone to rollover accidents. According to NHTSA, of the nearly 9.1 million passenger car, SUV, pickup and van crashes in 2010, only 2.1% involved a rollover, however, rollovers accounted for nearly 35% of all deaths [1]. Rollover has become a severe challenge to vehicle safety.

Vehicle rollover can be divided into two types: tripped and un-tripped. Tripped rollover usually occurs, when a vehicle leaves the roadway and slides sideways, digging its tires into soft soil or striking an object such as a curb or guardrail. Instead of an object serving as a tripping mechanism, un-tripped rollover usually occurs during high-speed collision avoidance maneuvers, and mostly top-heavy vehicles [2]. Researches about rollover mostly focus on un-tripped rollover. C.B.Winkler et al. [4] analyzed lots of rollover accidents, found that heavy vehicles were more prone to rollover accidents, because they had high center of gravity, long body, and a relatively narrow track, leading to lower static rollover threshold than light vehicles. However, rollovers happen dynamically, and Bernard J et al. [5] found that vehicles roll over at a lower threshold than the static. Erik Dahlberg [6] proposed the concept of dynamic rollover threshold, the least lateral acceleration resulting in rollover without influences of external forces, which may be different for different types of vehicles. Jangyeol Yoon and Kyongsu Yi [7] also presented a dynamic rollover index to indicate rollover by a roll dynamics phase plane analysis. Cooperrider et al. [8] investigated actual rollover conditions, found that the quicker lateral acceleration increased, the shorter time the vehicle took to roll over, when the lateral acceleration exceeded static rollover threshold. After Preston-Thomas and Woodrooffe [9] used lateral load transfer ratio (LTR), which was first proposed by R. D. Ervin [3], as their rollover warning device's threshold, rollover researches were carried forward to practical usage in rollover prediction and prevention. Many researchers presented different LTRs as indicators in their anti-rollover systems [10-15].

Rollover is dealing with extreme safety situation, so finding out the critical rollover point is crucial in predicting rollover accident. We present in this paper an improved novel generalized threshold model based on LTR. Unlike direct bi-axial vehicle models in most researches, we build a multi-axial vehicle rollover model to get a universal threshold, considering the characteristics of vehicle suspension and limit equilibrium of tires' lifting-off. Furthermore, under some assumptions, we develop a more practical and generalized LTR, called GLTR. The GLTR is determined by roll angle and roll angular rate together with some vehicle parameters. In order to test the performances of generalized LTR, two other commonly used LTR formulas [11, 12] are compared and investigated both on the rollover warning platform and TruckSim platform.

2 Dynamic rollover thresholds

2.1 LTR as dynamic rollover threshold

Dynamic rollover is closely related to load transfer ratio (LTR), which can be taken as rollover indicator to evaluate vehicles' dynamic rollover stability.

R. D. Ervin [3], in 1986, first proposed the definition of LTR (DLTR) and then Preston and Woodrooffe [9] used it in their initial rollover warning device, as in (1). If there is no lateral load transfer, DLTR is zero; if the lateral load transfers to another side totally, DLTR is ± 1.

$$DLTR = \sum_{i=1}^{n}(F_{Li} - F_{Ri}) / \sum_{i=1}^{n}(F_{Li} + F_{Ri}) \tag{1}$$

Many researchers began to use LTR, the normalized indicator. The following two different LTR formulas are commonly used rollover indicators, which can be compared with our GLTR.

2.2 Reference rollover thresholds

With consideration of roll motion and roll moment, S. Selim et al. [11] proposed a dynamic rollover indicator, as shown below.

$$LTR_1 = -2(k\phi + c\dot{\phi}) / mgT \tag{2}$$

Derived from roll dynamics, the rollover estimation in (2) can detect the transient phase of rollover. However, some authors argued that this is also not sufficient to estimate the rollover since the lateral dynamics, which is a critical factor in rollover, is ignored in the formula. Furthermore, Hsun-Hsuan Huang's rollover model [12] includes the lateral acceleration and roll dynamics simultaneously, as in (3).

$$LTR_2 = -2(k\phi + c\dot{\phi} + m_s a_{y,2} h_R) / mgT \tag{3}$$

Cooperrider et al. [8] found that the quicker lateral acceleration increased, the shorter time the vehicle took to roll over, which means there's still an additional rolling force taking the vehicle to rollover quickly, breaking the dynamic balance of tires' lifting-off. Besides, un-tripped rollover occurs mostly on heavy load vehicles, in which multi-axial vehicles take up an increasing percentage. Therefore, in what follows, we present an improved generalized rollover threshold GLTR to solve the problem.

2.3 GLTR for multi-axial vehicles

2.3.1 Complicated rollover threshold

According to the formulas above, we can figure out bi-axial vehicles' dynamic rollover threshold by measuring LTR. No matter bi-axial or multi-axial vehicle, when it's steering or doing other curvilinear motions, the vehicle body will roll under the action of the lateral force and the vehicle's center of gravity will tilt to the outside leading to increased lateral movement. When the inside of the vehicle loses support, it starts to roll. However, generalized multi-axial vehicles' rollover threshold can be quite different, because each axle's lateral load transfer should be considered. A schematic of the vehicle rollover model is shown in Figure 1. Center of gravity is a virtual point of the vehicle body, which is shown in the figure as an inclined rectangle. And some assumptions should be mentioned: firstly, the frame is a rigid frame, ignoring the body deformation; secondly, the vehicle's unsprung mass is ignored; finally, roll angle of each axis is equal.

Considering the force and moment balance in the roll movement, vehicle mass can be approximately regarded as the sprung mass because the unsprung mass of the suspension is relatively less than the sprung mass above the body. If the deformation of the vehicle body is ignored and the roll angle is assumed the same for each axis, then the roll moment balance acting on the vehicle body is as follows.

$$\sum_{j=1}^{n} K_{\phi,j}\phi_j + \sum_{j=1}^{n} C_{\phi,j}\dot{\phi}_j = m_s a_y h_s + m_s h_s g\phi \quad (4) \Rightarrow a_y = (k\phi + c\dot{\phi} - m_s h_s g\phi) / m_s h_s \qquad (5)$$

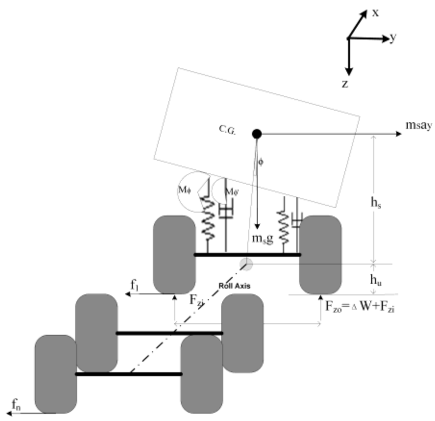

Figure 1. Rollover model of multi-axial vehicle

The wheels' lateral forces and the vehicle's inertia forces are balanced as shown in Figure 2.

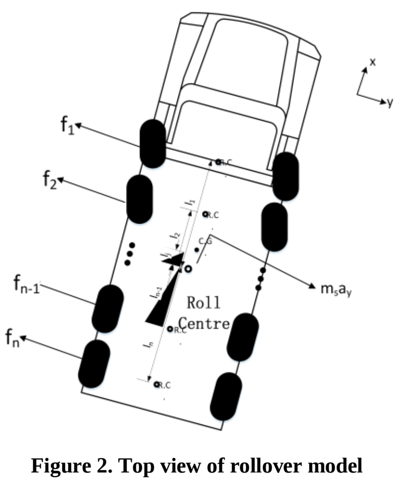

Figure 2. Top view of rollover model

The equation below is obtained from lateral force balance.

$$\sum_{j=1}^{n} f_j = m_s a_y \Rightarrow f_j = m_s a_y / (l_j \sum_{i=1}^{n} \frac{1}{l_i}) \qquad (6)$$

When in curvilinear motion, ΔW_j, load of each axis transfers to the outside, assuming the vehicle track is the same, then

$$\Delta W_j \cdot \frac{T}{2} = f_j h_{u,j} + K_{\phi,j}\phi_j + C_{\phi,j}\dot{\phi}_j \tag{7}$$

The load transfer of each axis is revealed from (5), (6) and (7). According to LTR definition in (1), we can see below.

$$\begin{aligned} LTR = \frac{\sum_{j=1}^{n} \Delta W_j}{mg} &= \sum_{j=1}^{n} \frac{-2\left(c\frac{h_{u,j}}{h_s} + C_{\phi,j} l_j \sum_{i=1}^{n} \frac{1}{l_i} \right)\dot{\phi}_j}{mgT_j l_j \sum_{i=1}^{n} \frac{1}{l_i}} \\ &+ \sum_{j=1}^{n} \frac{-2\left((k - m_s g h_s)\frac{h_{u,j}}{h_s} + K_{\phi,j} l_j \sum_{i=1}^{n} \frac{1}{l_i} \right)\phi_j}{mgT_j l_j \sum_{i=1}^{n} \frac{1}{l_i}} \end{aligned} \tag{8}$$

We take the new derivation of LTR above as dynamic rollover threshold of multi-axial vehicles. However, it's somehow too complicated to use in practical scenarios. Therefore, in what follows, we try to find the simple form of multi-axial vehicles' LTR, by using some model simplifying assumptions.

2.3.2 Generalized rollover threshold

We make some additional assumptions to simplify the model as follows: firstly, the vehicle's wheelbase is equal; secondly, roll center of the vehicle for each axis is at the same height from the ground, forming a level roll axis of the vehicle. The simplified model is shown in Figure 3.

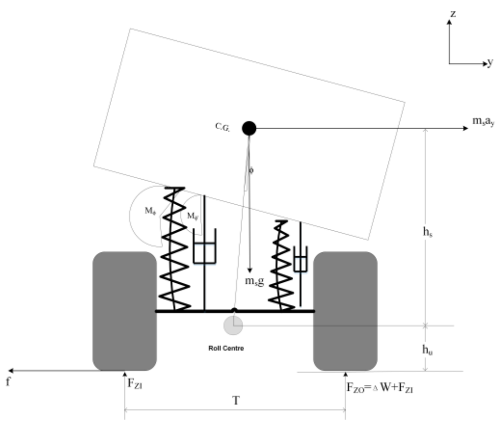

Figure 3. Back view of simplified rollover model

From the assumptions above (shown in Fig. 3), we know that

$$T = T_j; h_u = h_{u,j}; \sum_{j=1}^{n} \frac{1}{l_j \sum_{i=1}^{n} \frac{1}{l_i}} = 1 \tag{9}$$

By combining (8) and (9), we can derive the simplifying and generalized LTR below.

$$GLTR = -\frac{2\left((h_u + h_s)c\dot{\phi} + (h_u + h_s)k\phi - m_s g h_s h_u \phi\right)}{mgh_s T} \tag{10}$$

From above, we know the novel derivation in (10) is much more generalized than (8), as well as simple. After some specific assumptions and simplifications, the dynamic rollover threshold can be used for any multi-axial vehicle, or bi-axial vehicle.

3 Simulations

The TruckSim software is used to verify GLTR performance in our rollover detection study. TruckSim is a professional vehicle system simulation program developed by University of Michigan Transportation Research Institute (UMTRI). TruckSim has been commercialized and can be licensed from the Mechanical Simulation Corporation (MSC). TruckSim is capable of simulating and visualizing the full nonlinear vehicle dynamic response of single unit vehicles with at most 4 axles, so we can configure the 4-axle vehicle model in TruckSim and acquire the vehicle status under different procedures.

3.1 TruckSim Platform

It's hard to get a real vehicle to rollover, but it's easy to make a vehicle rollover truly in simulations. We build a TruckSim platform as below to verify the proposed GLTR. In Figure 4, a 4-axle vehicle model is configured appropriately, we set the working conditions of step steer and fishhook in drive controls, and then we can get the vehicle responses, which can be used to compare GLTR and other thresholds.

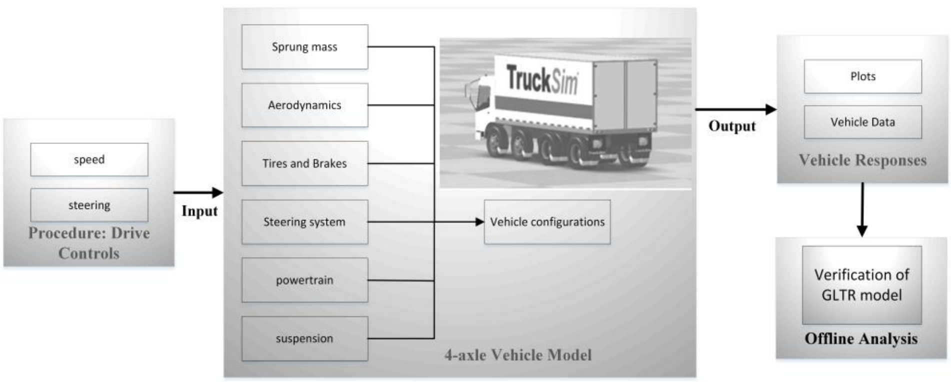

Figure 4. TruckSim platform for a 4-axle vehicle

TruckSim platform is professional in providing all vehicle responses, especially all the vertical tire forces, from which we can figure out the true LTR in definition of (1).

Table 1. Intrinsic parameters of 4-axle vehicle

name	value	name	value
m_s	21585 kg	g	9.8 m/s^2
m_u	1000 kg	T	1950 mm
h_u	528 mm	k	96762 Nm/deg
h_s	872 mm	c	1400 Nm/(deg s)

The 4-axle simulation vehicle is configured as seen in Table 1, and we set the working conditions of step steer and fishhook in drive controls. Furthermore, we set GLTR=±0.6 as threshold of warning signal for offline data analysis. The warning signal is in red solid line in the following figures.

3.2 Simulations results

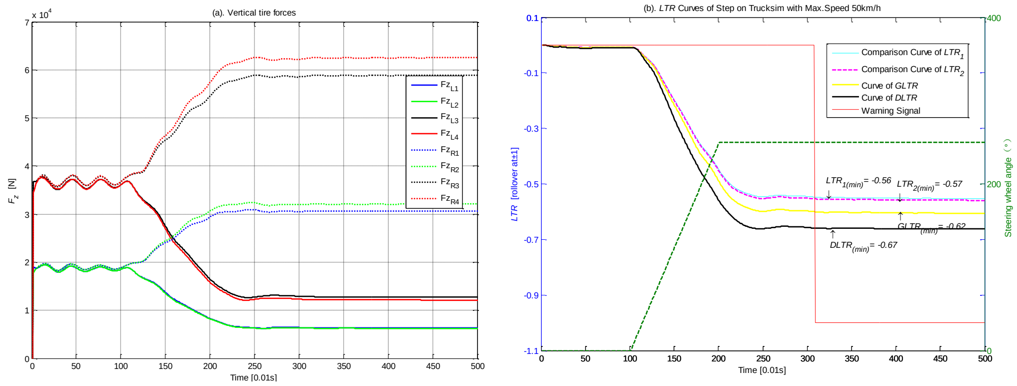

Figure 5. Simulation results of step without rollover

3.2.1 Step results

We have contrast simulation results of step motion, with no rollover and with rollover occurring, as shown in Figure 5 and 6. The steering wheel angle input can be seen in green dash line. On TruckSim platform we can get actual load transfers, so as to acquire the DLTR (in black solid line), which can be taken as criterion for other three LTR models.

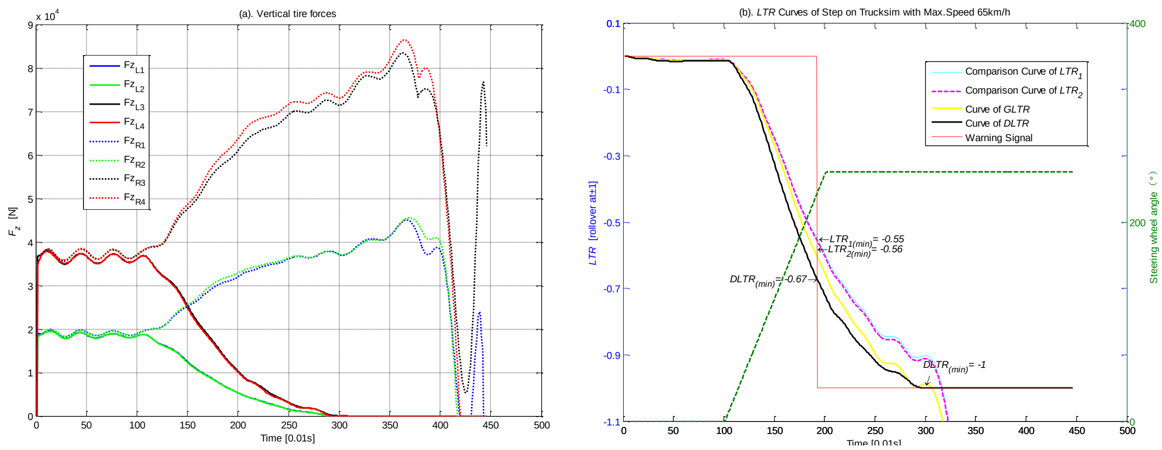

Figure 6. Simulation results of step with rollover occurring

Figure 5 and 6 show the simulation results at two different maximum speeds: Figure 5 at 50km/h without rollover happening and Figure 6 at 65km/h with rollover occurring. Under different models and formulas (Eq. (1), (2), (3) and (10)), we get four different LTR curves.

From Figure 5(a) below, the curves of vertical tire forces, we can see the simulation vehicle is turning left, and vertical loads transfer to the right side, so their vertical tire forces are all larger than the left side.

While the lower one tells that, all the curves of LTR have similar trends. LTR1 is close to LTR2, but smaller than it; GLTR is close to DLTR, but smaller than it, too; both LTR1 and LTR2 are smaller than GLTR and DLTR. The extreme values of LTR1, LTR2, GLTR and DLTR are -0.56, -0.57, -0.62 and -0.67 respectively.

From Figure 6, we can see the vertical loads transfer to the right side totally, which means the simulation vehicle get rollover eventually. When GLTR reaches -0.6, the trigger value of warning signal, the values of LTR_1, LTR_2 and DLTR are respectively -0.55, -0.56 and -0.67. Obviously, in the case that rollover is occurring, namely DLTR=-1, the LTRs increased over unity suddenly. But GLTR, closest to DLTR, can still indicate the impending rollover more accurate than LTR_1 and LTR_2.

3.2.2 Fishhook results

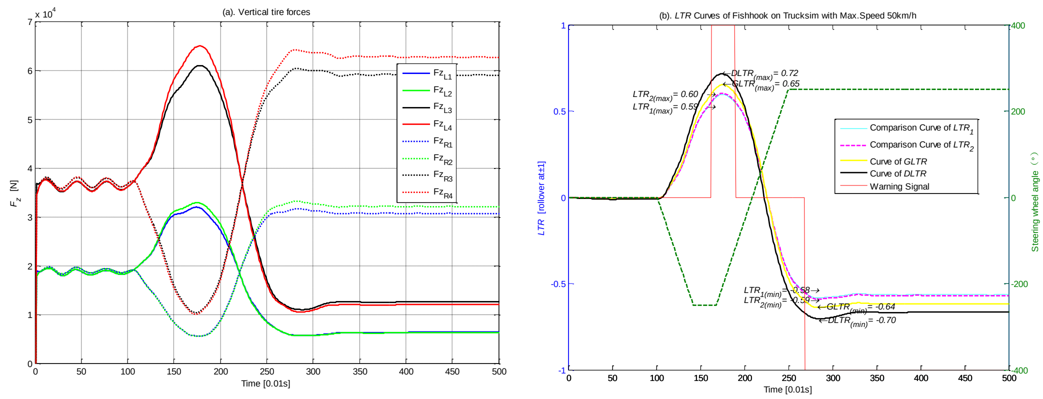

Figure 7. Simulation results of fishhook without rollover

Fishhook is a quite intense driving behavior. We have contrast simulation results of fishhook, as shown in Figure 7 and 8. The former one is a working condition without rollover, while the latter one is a rollover case of fishhook.

From Figure 7(a), we know the 4-axle simulation vehicle is safe without rollover, but still it triggered the warning signals twice, as the absolute value of GLTR exceeds 0.6 seen in Figure 7(b). The situation of what Figure 7(b) shows, is that all curves have similar trends and GLTR is close to DLTR, but larger than LTR1 and LTR2 as a whole.

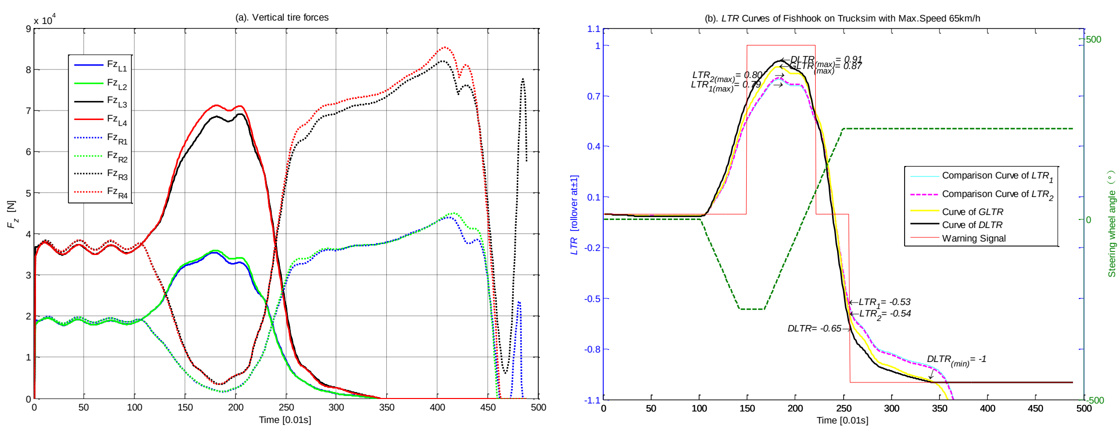

Figure 8. Simulation results of fishhook with rollover occurring

We can also find that in the 1st turn of fishhook, the extreme values of LTR1, LTR2, GLTR and DLTR are respectively 0.59, 0.60, 0.65 and 0.72. While in the 2nd turn of fishhook, the extreme values of LTR1, LTR2, GLTR and DLTR are -0.58, -0.59, -0.64 and -0.70 respectively.

From Figure 8(a), we can see the vertical loads eventually transfer to the left side totally when the value of DLTR approaches -1, which means the simulation vehicle gets rollover in the end.

As we have set the proportion of GLTR to 0.6 as rollover warning threshold (in red solid line). It is a conservative threshold, but we can see clearly how the three LTRs change. Before rollover's coming, when the actual DLTR is -0.65, we send out the warning signal by GLTR's value -0.60, while the other values of LTR1 and LTR2 are -0.53 and -0.54, respectively. We can infer that GLTR is more accurate than LTR1 and LTR2.

4 Experiments

Although it's difficult and dangerous to get the vehicle to rollover in real tests, verification experiments on test vehicle are much more persuasive than simulations. To insure the safety of rollover, we design a test scheme of our 2-axle test vehicle with real-time RWP installed on.

4.1 RWP based on GLTR

RWP is designed based on GLTR to give drives real-time warning signals of rollover. From (10), we know that only two variables are required to compute GLTR, but lots of other intrinsic parameters of vehicle should be known. On the platform shown in Figure 9, we use an acquisition module such as RT3000 to obtain critical parameters and variables, a computing module to figure out the real-time LTR value so as to judge the rollover danger level, a warning module to alarm the danger, and a memory module to store data for certain offline data analysis to testify the rollover model and make some adjustments.

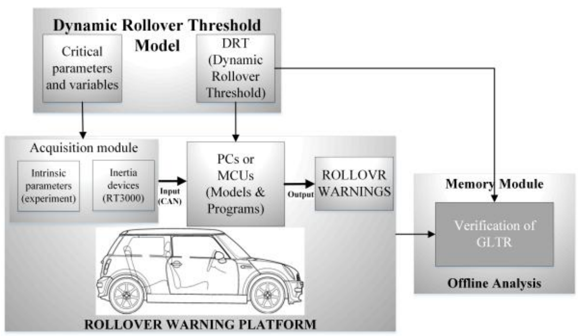

Figure 9. RWP based on GLTR

Table 2. Parameters required by GLTR of test vehicle

name	value	name	value
m_s	1585 kg	g	9.8 m/s^2
m_u	175 kg	T	1540 mm
h_u	90 mm	k	1873 Nm/deg
h_s	449 mm	c	24 Nm/(deg s)

As seen in Figure 9, the RWP can be installed on the test vehicle and parameters shown below in Table 2 are employed to figure out GLTR.

4.2 Test scenes

According to the national standard and the test evaluation method on the vehicle steering stability, we have performed step and snake tests. In actual experiments, we had enough times of the step and snake tests, trying to push the tests to the limit on the premise of safety. During those experiments, observers could even see one of the vehicle tires lifting off the road, as shown in Figure 10.

Figure 10. Actual test scenes

4.3 Test results

Step and snake tests are intense driving, and we can investigate the values of LTR in Table 3 and Table 4 about the test scenes. Because we have RWP installed on the test vehicle, and to alert the driver immediately we set the proportion of our RWP's GLTR to ±0.6 as rollover warning threshold. If GLTR is bigger than 0.6 or smaller than -0.6, warning signal is set to 1 or -1, giving out the rollover warning alarm. Otherwise, warning signal is zero, the alarm is quiet and the test vehicle is safe.

4.3.1 Step results

Table 3. Extreme values in step tests

No.	LTR_1	LTR_2	GLTR	Max a_y	Max V_x
1	-0.41	-0.47	-0.49	-5.4 m/s^2	35.1 km/h
2	-0.50	-0.58	-0.59	-7.6 m/s^2	41.4 km/h
3	-0.59	-0.69	-0.70	-9.7 m/s^2	47.8 km/h

As seen in Table 3, step tests were carried out turning left with different maximum speeds, so the values of LTR are all negative, with the lateral load transferring to the right side of the vehicle. With the speed and lateral acceleration increasing, all the values of LTR increase, but LTR1 is smaller than LTR2, and LTR2 closer but smaller than GLTR. In the 3rd step test, with the lateral acceleration approaching 1g, both LTR2 and GLTR will trigger the warning alarm in RWP, whose threshold is ±0.6, but LTR1 will not.

The 3rd of the step results was shown below to clearly compare those three LTRs and display the test vehicle's rolling trends. From Figure 11, we can see the red warning line jumping to -1, so we know the warning signal is triggered by GLTR. Comparing the trends and peaks of three curves of different LTRs, we also know that though LTR_2 and GLTR thought it a danger for the vehicle to rollover, LTR_1 thought it safe. So if it is really a danger, the rollover index LTR1 will not trigger any warning alarms.

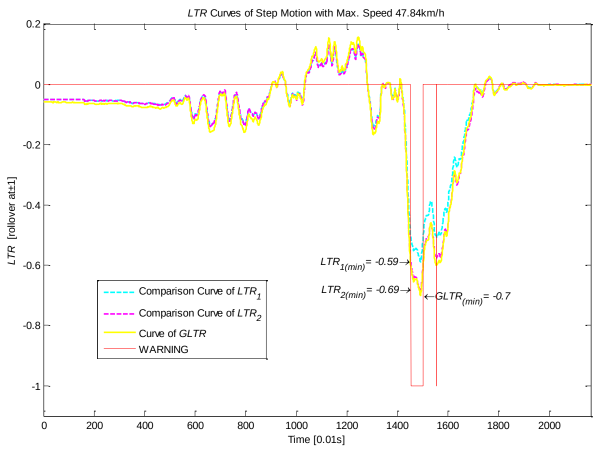

Figure 11. Test results of step with warning signal triggered

4.3.1 Snake results

Table 4. Extreme values in snake tests

No.	LTR_1	LTR_2	GLTR	Max a_y	Max V_x
1	-0.49/ +0.47	-0.55/ +0.54	-0.58/ +0.56	-7.0/+7.0 m/s^2	61.0 km/h
2	-0.47/ +0.51	-0.53/ +0.58	-0.55/ +0.61	-6.2/+7.3 m/s^2	62.1 km/h
3	-0.59/ +0.55	-0.65/ +062	-0.68/ +0.65	-8.1/+7.4 m/s^2	60.4 km/h

Unlike step tests, snake motion moves to both sides, so results shown in Table 4 indicate both positive and negative trends of the vehicle. The three snake tests in Table 4, are repeated trials. Real tests are not like simulations, and each test is kind of different from the others. However, we can still find the similar disciplines as in Table 3. LTR1 is smaller than LTR2, and LTR2 is closer but smaller than GLTR.

To be more clear about the process of snake test, we select the 1st of the snake results shown in Figure 12.

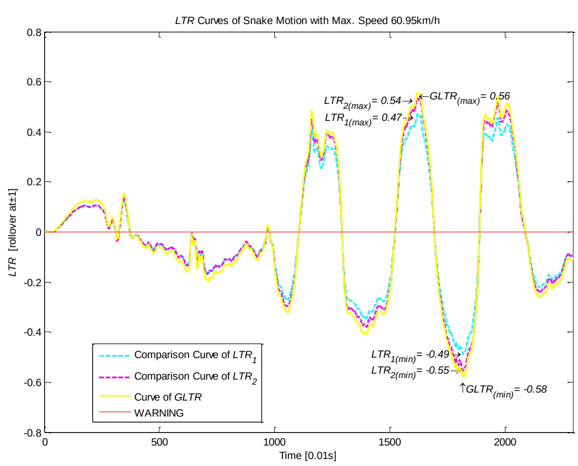

Figure 12. Test results of snake with warning signal triggered

As seen in Figure 12, the snake test shows the similar trends as in step tests, and still, we can see GLTR is the most sensitive rollover index. The red line is zero in Figure 12, so the warning alarm on RWP is not triggered by GLTR. All the results show that our GLTR could just approach 0.70 in actual tests on our test vehicle. But it's already a quite high value of LTR, without anti-roll bar equipped on the vehicle. For real tests, the results are enough to verify the performance of GLTR based on our novel rollover model.

5 Conclusions

This paper has presented an improved dynamic rollover model of multi-axial vehicle based on LTR. A novel dynamic rollover threshold, namely GLTR, which indicates the impending rollover status, has been proposed. A real-time RWP based on GLTR, has been designed to warn drivers of rollover risk. Simulations on TruckSim are performed to verify reliability of the true rollover prediction by those LTR models. Moreover, by comparison of GLTR and two other LTR models on RWP, the sensitivity and practicality of LTR are verified. Results show that GLTR presented in this paper is a promising rollover indicator, and can efficiently and actually indicate the impending rollover.

In addition, by studying the rollover curves of different driving conditions, the characteristics of LTR in rollover can be found. In the case of no rollover, the trends of LTRs are similar and maintained around unity; while in the case that rollover is occurring, the LTRs increased over ±1 sharply. To avoid rollover, it's reasonable to use the most accurate LTR as dynamic rollover threshold, and set a conservative threshold of the LTR.

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Nomenclature

φ roll angle of sprung mass
$\dot{\varphi}$ roll angular rate of sprung mass
h height of center of sprung mass, measured upwards from the ground

h_s	distance of center of sprung mass, measured from roll axis of the vehicle
$h_{s,j}$	distance of center of sprung mass, measured from roll center of the jth axle
$h_u (h_R)$	height of roll axis, measured upwards from the ground
$h_{u,j}$	height of roll center of the jth axle, measured upwards from the ground
T	track width
T_j	track width of the jth axle or suspension
m_s	sprung mass
$a_y (a_{y,s})$	lateral acceleration of sprung mass
g	acceleration of gravity
k	total torsional spring stiffness
$K_{\varphi,j}$	torsional spring stiffness of the jth axle
c	total torsional damper coefficients
$C_{\varphi,j}$	torsional damper coefficient of the jth axle
f_j	lateral force of the jth axle, balanced with the corresponding inertia force
l_j	longitudinal distance to the jth axle, measured forwards from the center of total mass
F_L	vertical supporting forces of left wheels
F_R	vertical supporting forces of right wheels
F_{ZI}	vertical supporting forces of inward wheels
F_{ZO}	vertical supporting forces of outward wheels
ΔW	load transfer of the vehicle
ΔW_j	load transfer of the jth a