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Devising a Method to Calculate the Trajectory of a Dummy's Head Using Linear Acceleration and Angular Velocity

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Abstract: The method of calculating the head trajectory in three dimensions within the passenger compartment using the head and vehicle's acceleration and angular velocity in their respective local coordinate systems was constructed. To verify this calculation method, a crash simulation in LS-DYNA was conducted to compare the calculated and simulated result. Next, in order to demonstrate real-world usability of this method, the dummy's head trajectory was calculated using the linear acceleration and angular velocity of the dummy's head obtained from a sled test. The calculated result was then compared to the trajectory obtained from video analysis of the crash test. The possibility of obtaining the head trajectory in the global coordinate system from the local head's acceleration and angular velocity using LS-DYNA and MADYMO was also investigated. The head trajectory of a vehicle occupant with respect to the vehicle coordinate system and global coordinate system can be obtained using these methods.

Keywords: Trajectory, Euler angle, Transformation matrix

1 Introduction

It is important that head trajectory within the passenger compartment is obtained during a crash test. This will allow for the analysis of the interaction between the head and the airbag, as well as the head contact with the various parts of the passenger compartment.

For example, in the oblique MDB test as well as the small overlap test, the head moves towards the direction of the A-pillar due to the rotation of the passenger compartment. Knowing this head motion can lead to a better understanding of the kinematics of the crash.

Shea and Viano [1] have shown that the two-dimensional head trajectory of a dummy can be obtained by first calculating the angle of the head's rotation with respect to the global coordinate system using the angular velocity sensor attached in the Hybrid III dummy's head, and then using this angle to convert the dummy's head acceleration components from the head's local coordinate system to the global coordinate system.

Park et al. [2] mentioned that the three-dimensional head trajectory including rotation can be obtained using the head's acceleration and angular velocity in its local coordinate system. However, the methodology was not written in their paper.

It is also not possible to obtain the head posture solely from the head acceleration, thus the three-dimensional head trajectory also cannot be obtained.

On the other hand, in recent times, an angular velocity sensor has been attached onto the dummy's head in order to calculate brain injury criteria (BrIC) from the head's rotation. Using this angular velocity sensor, the head posture can thus be obtained.

In this research, the method of calculating the head trajectory within the passenger compartment in three dimensions using the head and the vehicle's acceleration and angular velocity in their respective local coordinate systems was constructed and validated by simulation. Furthermore, the possibility of obtaining the head trajectory using LS-DYNA and MADYMO was also investigated.

2 Method of determining trajectory of the head using Euler angle

Accelerometers are attached to the dummy's body such as the head, thorax and lumbar. During a vehicle crash test, the dummy's motion will be both translational as well as rotational. The measurements obtained from the accelerometer are the acceleration components projected on each of the axes of the accelerometer's local coordinate system, which is in other words the moving coordinate system attached onto the dummy's body. Therefore, it is not possible to obtain the correct velocity (v_x, v_y, v_z) and trajectory (X, Y, Z) of the head in the global coordinate system directly by merely integrating the acceleration components recorded by the accelerometer.

This section introduces the method of determining the head acceleration, velocity and displacement within the passenger compartment once the acceleration and the angular velocity of the head in its local coordinate system are obtained.

First step would be to define the global coordinate system O-XYZ, the vehicle coordinate system O_0-$x'y'z'$ and the dummy's head coordinate system O_1-xyz (Figure 1).

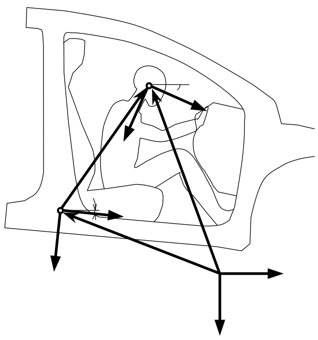


Figure 1. Global coordinate system, and the local coordinate system of the passenger compartment and dummy's head

2.1 Transformation of the coordinate system

The measurements obtained from the accelerometer are the acceleration of the head $\boldsymbol{a}$ projected onto each axis of the head's local coordinate system O_1-xyz (Figure 2). The unit vectors of the global coordinate $\boldsymbol{e}_X$, $\boldsymbol{e}_Y$, $\boldsymbol{e}_Z$ and the local coordinate system $\boldsymbol{e}_x$, $\boldsymbol{e}_y$, $\boldsymbol{e}_z$ are defined (Figure 3). Equation (1) shows that it is possible to represent the acceleration vector $\boldsymbol{a}$ as a function of the acceleration components in either the global coordinate system (a_X, a_Y, a_Z) or the local coordinate system (a_x, a_y, a_z).

$$\boldsymbol{a} = a_X\boldsymbol{e}_X + a_Y\boldsymbol{e}_Y + a_Z\boldsymbol{e}_Z = a_x\boldsymbol{e}_x + a_y\boldsymbol{e}_y + a_z\boldsymbol{e}_z \tag{1}$$

The direction of the local coordinate system changes with respect to the dummy's movement, while the global coordinate system will always remain the same. The components of the local acceleration of the head a_x, a_y and a_z are obtained from the projection of the acceleration of the head $\boldsymbol{a}$ onto the head's local coordinate system.

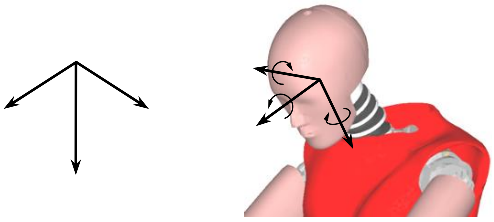


Figure 2. Detailed view of the global coordinate system and the dummy's head local coordinate system

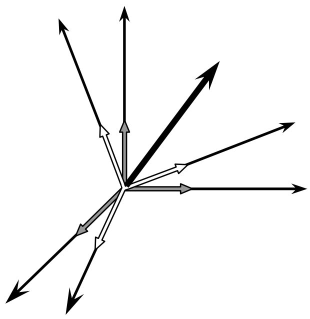


Figure 3. Base vectors of the global coordinate system and dummy's head local coordinate system

Equation (2) represents the relationship between the unit vectors of the global and local coordinate system using the direction cosine of the angle between each unit vectors.

$$\begin{aligned}\boldsymbol{e}_x &= (\boldsymbol{e}_x \cdot \boldsymbol{e}_X)\boldsymbol{e}_X + (\boldsymbol{e}_x \cdot \boldsymbol{e}_Y)\boldsymbol{e}_Y + (\boldsymbol{e}_x \cdot \boldsymbol{e}_Z)\boldsymbol{e}_Z \\ \boldsymbol{e}_y &= (\boldsymbol{e}_y \cdot \boldsymbol{e}_X)\boldsymbol{e}_X + (\boldsymbol{e}_y \cdot \boldsymbol{e}_Y)\boldsymbol{e}_Y + (\boldsymbol{e}_y \cdot \boldsymbol{e}_Z)\boldsymbol{e}_Z \\ \boldsymbol{e}_z &= (\boldsymbol{e}_z \cdot \boldsymbol{e}_X)\boldsymbol{e}_X + (\boldsymbol{e}_z \cdot \boldsymbol{e}_Y)\boldsymbol{e}_Y + (\boldsymbol{e}_z \cdot \boldsymbol{e}_Z)\boldsymbol{e}_Z\end{aligned} \tag{2}$$

Equation (3) is the result of the dot product of $\boldsymbol{a}$ from Equation (1) and $\boldsymbol{e}_x$.

$$\begin{aligned}\boldsymbol{a} \cdot \boldsymbol{e}_x &= a_X(\boldsymbol{e}_X \cdot \boldsymbol{e}_x) + a_Y(\boldsymbol{e}_Y \cdot \boldsymbol{e}_x) + a_Z(\boldsymbol{e}_Z \cdot \boldsymbol{e}_x) \\ &= a_x(\boldsymbol{e}_x \cdot \boldsymbol{e}_x) + a_y(\boldsymbol{e}_y \cdot \boldsymbol{e}_x) + a_z(\boldsymbol{e}_z \cdot \boldsymbol{e}_x) \\ &= a_x\end{aligned} \tag{3}$$

Applying this relationship to the other components, the relationship between the two acceleration vectors represented by each coordinate system is obtained.

$$\begin{aligned}\begin{Bmatrix} a_x \\ a_y \\ a_z \end{Bmatrix} &= \begin{bmatrix} \boldsymbol{e}_x \cdot \boldsymbol{e}_X & \boldsymbol{e}_x \cdot \boldsymbol{e}_Y & \boldsymbol{e}_x \cdot \boldsymbol{e}_Z \\ \boldsymbol{e}_y \cdot \boldsymbol{e}_X & \boldsymbol{e}_y \cdot \boldsymbol{e}_Y & \boldsymbol{e}_y \cdot \boldsymbol{e}_Z \\ \boldsymbol{e}_z \cdot \boldsymbol{e}_X & \boldsymbol{e}_y \cdot \boldsymbol{e}_Y & \boldsymbol{e}_z \cdot \boldsymbol{e}_Z \end{bmatrix} \begin{Bmatrix} a_X \\ a_Y \\ a_Z \end{Bmatrix} = [A] \begin{Bmatrix} a_X \\ a_Y \\ a_Z \end{Bmatrix} \\ \begin{Bmatrix} a_X \\ a_Y \\ a_Z \end{Bmatrix} &= \begin{bmatrix} \boldsymbol{e}_X \cdot \boldsymbol{e}_x & \boldsymbol{e}_X \cdot \boldsymbol{e}_y & \boldsymbol{e}_X \cdot \boldsymbol{e}_z \\ \boldsymbol{e}_Y \cdot \boldsymbol{e}_x & \boldsymbol{e}_Y \cdot \boldsymbol{e}_y & \boldsymbol{e}_Y \cdot \boldsymbol{e}_z \\ \boldsymbol{e}_Z \cdot \boldsymbol{e}_x & \boldsymbol{e}_Z \cdot \boldsymbol{e}_y & \boldsymbol{e}_Z \cdot \boldsymbol{e}_z \end{bmatrix} \begin{Bmatrix} a_x \\ a_y \\ a_z \end{Bmatrix} = [A]^T \begin{Bmatrix} a_x \\ a_y \\ a_z \end{Bmatrix}\end{aligned} \tag{4}$$

$[A]$ is the coordinate transformation matrix.

$$[A] = \begin{bmatrix} \boldsymbol{e}_x \cdot \boldsymbol{e}_X & \boldsymbol{e}_x \cdot \boldsymbol{e}_Y & \boldsymbol{e}_x \cdot \boldsymbol{e}_Z \\ \boldsymbol{e}_y \cdot \boldsymbol{e}_X & \boldsymbol{e}_y \cdot \boldsymbol{e}_Y & \boldsymbol{e}_y \cdot \boldsymbol{e}_Z \\ \boldsymbol{e}_z \cdot \boldsymbol{e}_X & \boldsymbol{e}_y \cdot \boldsymbol{e}_Y & \boldsymbol{e}_z \cdot \boldsymbol{e}_Z \end{bmatrix} \tag{5}$$

This matrix is the orthogonal matrix meaning $[A]^{-1}= [A]^T$ as shown from the two equations of Equation (4). Using the direction cosine of each coordinate axes and equation (5), it is possible to transform the components represented by two coordinate system.

Since Equation (4) represents the transformation of the vector component, in the same way the transformation of the relationship between the force components in global coordinate system (f_X, f_Y, f_Z) and local coordinate system (f_x, f_y, f_z) can be also represented by Equation (4).

It is important to remember that since the dummy's local coordinate system O_1-xyz rotates with time, the unit vectors also rotate with time. Equation (6) shows that the velocity vector can be represented as a function of the unit vectors $\boldsymbol{e}_x$, $\boldsymbol{e}_y$, $\boldsymbol{e}_z$ and components v_x, v_y, v_z, obtained by projecting the velocity vector onto the axes of the O_1-xyz coordinate system.

$$\boldsymbol{v} = v_x\boldsymbol{e}_x + v_y\boldsymbol{e}_y + v_z\boldsymbol{e}_z \tag{6}$$

Hence, by differentiating the above equation (Equation 6), the acceleration $\boldsymbol{a}$ can be obtained. Using the angular velocity vector $\boldsymbol{\omega}$, the time variation of unit vectors $\boldsymbol{e}_x$, $\boldsymbol{e}_y$, $\boldsymbol{e}_z$ can be represented by $d\boldsymbol{e}_x/dt=\boldsymbol{\omega}\times\boldsymbol{e}_x$, $d\boldsymbol{e}_y/dt=\boldsymbol{\omega}\times\boldsymbol{e}_y$, $d\boldsymbol{e}_z/dt=\boldsymbol{\omega}\times\boldsymbol{e}_z$. Therefore, the acceleration is as shown in Equation (7).

$$\begin{aligned}\boldsymbol{a} &= \frac{d\boldsymbol{v}}{dt} = \frac{dv_x}{dt}\boldsymbol{e}_x + \frac{dv_y}{dt}\boldsymbol{e}_y + \frac{dv_z}{dt}\boldsymbol{e}_z + \boldsymbol{\omega}\times(v_x\boldsymbol{e}_x + v_y\boldsymbol{e}_y + v_z\boldsymbol{e}_z) \\ &= \frac{dv_x}{dt}\boldsymbol{e}_x + \frac{dv_y}{dt}\boldsymbol{e}_y + \frac{dv_z}{dt}\boldsymbol{e}_z + \boldsymbol{\omega}\times\boldsymbol{v} \qquad (7)\end{aligned}$$

Equation (8) now shows each components of above equation (Equation 7) when the angular velocity vector is represented as $\boldsymbol{\omega}=\omega_x\boldsymbol{e}_x+\omega_y\boldsymbol{e}_y+\omega_z\boldsymbol{e}_z$.

$$\begin{aligned} a_x &= \frac{dv_x}{dt} + \omega_y v_z - \omega_z v_y \\ a_y &= \frac{dv_y}{dt} + \omega_z v_x - \omega_x v_z \\ a_z &= \frac{dv_z}{dt} + \omega_x v_y - \omega_y v_x \end{aligned} \tag{8}$$

As seen from Equation (8), it is not possible to obtain the correct velocity components v_x, v_y, v_z if only the acceleration components in dummy's coordinate system are time-integrated directly without the angular velocity components.

For the three-dimensional movement of a rigid body, representing the posture of the rigid body is complicated because the local coordinate system that is embedded into the rigid body rotates. Using Euler angles, it is possible to transform a coordinate system which has the same orientation as the global coordinate system O-XYZ into any arbitrary local coordinate system O_1-xyz attached on a rigid body [3]. Figure 4 shows the transformation by using Euler angles. The coordinate system which has the same orientation as the global coordinate system undergoes three rotations, yaw with an angle ψ, pitch with an angle θ and roll with an angle φ in that particular order, in order to obtain the desired local coordinate system. The O-$X_1Y_1Z_1$ coordinate system is defined by rotating the O-XYZ coordinate system about the Z-axis by the yaw angle ψ.The transformation equation of the vector's components (Equation 9) is thus obtained from the direction cosine of each of the coordinate axes.

$$\begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \end{Bmatrix} = R_Z(\psi)\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} \tag{9}$$

Next, the O-$X_2Y_2Z_2$ coordinate system is defined by rotating the O-$X_1Y_1Z_1$ coordiate system about the Y_1 axis by the pitch angle θ. Finally, the O-$X_2Y_2Z_2$ coordinate system is rotated about the X_2 axis by the roll angle φ. The resulting coordinate system shows the direction O$_1$-xyz. Equation (10) shows the transformation of each of the coordinate system.

$$\begin{Bmatrix} X_2 \\ Y_2 \\ Z_2 \end{Bmatrix} = R_{Y_1}(\theta) \begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \end{Bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \end{Bmatrix}$$
$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = R_{X_2}(\phi) \begin{Bmatrix} X_2 \\ Y_2 \\ Z_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{Bmatrix} X_2 \\ Y_2 \\ Z_2 \end{Bmatrix} \tag{10}$$

Calculating those matrixes in that sequence, the transformation equation of vector components between O-XYZ and O$_1$-xyz is obtained.

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = R_{X_2}(\varphi) R_{Y_1}(\theta) R_Z(\psi) \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}$$
$$= \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi & \sin\varphi\sin\theta\sin\psi + \cos\varphi\cos\psi & \sin\varphi\cos\theta \\ \cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi & \cos\varphi\sin\theta\sin\psi - \sin\varphi\cos\psi & \cos\varphi\cos\theta \end{bmatrix} \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}$$
$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi & \cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi \\ \cos\theta\sin\psi & \sin\varphi\sin\theta\sin\psi + \cos\varphi\cos\psi & \cos\varphi\sin\theta\sin\psi - \sin\varphi\cos\psi \\ -\sin\theta & \sin\varphi\cos\theta & \cos\varphi\cos\theta \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \tag{11}$$

The coordinate system O$_1$-xyz rotates at the same angular velocity $\boldsymbol{\omega}$ as the rigid because O$_1$-xyz is fixed on the rigid. Therefore, the angular velocity of the rigid is same as that of O$_1$-xyz. Equation (12) shows the angular velocity $\boldsymbol{\omega}$ of O$_1$-xyz represented by the unit vectors of O$_1$-xyz. Angular velocities about the x,y,z axes of O$_1$-xyz are defined ω_x, ω_y, ω_z respectively.

$$\boldsymbol{\omega} = \omega_x \boldsymbol{e}_x + \omega_y \boldsymbol{e}_y + \omega_z \boldsymbol{e}_z \tag{12}$$

It is also possible to represent $\boldsymbol{\omega}$ by the time variation of Euler angle $\dot{\psi}$, $\dot{\theta}$, $\dot{\varphi}$ using the procedures of the coordinate transformation mentioned above.

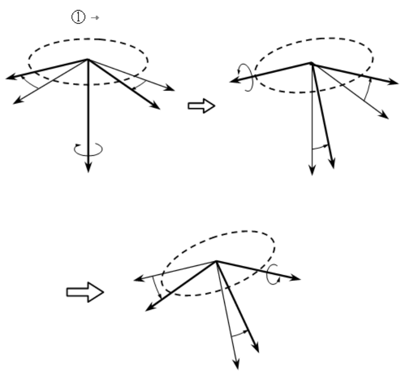


Figure 4. Transformation from the global coordinate system to the local coordinate system using Euler angles

$$\boldsymbol{\omega} = \dot{\psi}\,\boldsymbol{e}_Z + \dot{\theta}\,\boldsymbol{e}_{Y_1} + \dot{\varphi}\,\boldsymbol{e}_x \tag{13}$$

Referring to Equation (2) and Equation (4), the coordinate transformation matrix also represents the transformation of the unit vectors. Therefore, referring to the third row of the matrix in Equation (11), $\boldsymbol{e}_Z$ is represented as a function of $\boldsymbol{e}_x$, $\boldsymbol{e}_y$, $\boldsymbol{e}_z$ as shown in Equation (14).

$$\boldsymbol{e}_Z = -\sin\theta\,\boldsymbol{e}_x + \sin\varphi\cos\theta\,\boldsymbol{e}_y + \cos\varphi\cos\theta\,\boldsymbol{e}_z \tag{14}$$

With regard to $\boldsymbol{e}_{Y_1}$, from Equation (10) the coordinate transformation between O-$X_1Y_1Z_1$ and O_1-xyz is shown in Equation (15).

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = R_{X_2}(\varphi)\, R_{Y_1}(\theta) \begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \end{Bmatrix}$$

$$= \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ \sin\varphi\sin\theta & \cos\varphi & \sin\varphi\cos\theta \\ \cos\varphi\sin\theta & -\sin\varphi & -\cos\varphi\cos\theta \end{bmatrix} \begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \end{Bmatrix} \tag{15}$$

Hence, $\boldsymbol{e}_{Y_1}$ can be represented as shown in Equation (16) using the second column of the above coordinate transformation matrix.

$$\boldsymbol{e}_{Y_1} = \cos\varphi\,\boldsymbol{e}_y - \sin\varphi\,\boldsymbol{e}_z \tag{16}$$

Substituting $\boldsymbol{e}_Z$ and $\boldsymbol{e}_{Y_1}$ into Equation (13), the result is as shown in Equation (17).

$$\begin{aligned} \boldsymbol{\omega} &= (\dot{\varphi} - \dot{\psi}\sin\theta)\boldsymbol{e}_x + (\dot{\theta}\cos\varphi + \dot{\psi}\sin\varphi\cos\theta)\,\boldsymbol{e}_y \\ &+ (\dot{\psi}\cos\varphi\cos\theta - \dot{\theta}\sin\varphi)\,\boldsymbol{e}_z \end{aligned} \tag{17}$$

Therefore, Equation (18) shows the relationship between the Euler angle and angular velociy about the axes of O_1-xyz.

$$\begin{aligned} \omega_x &= \dot{\varphi} - \dot{\psi}\sin\theta \\ \omega_y &= \dot{\theta}\cos\varphi + \dot{\psi}\sin\varphi\cos\theta \\ \omega_z &= \dot{\psi}\cos\varphi\cos\theta - \dot{\theta}\sin\varphi \end{aligned} \tag{18}$$

Solving for $\dot{\psi}, \dot{\theta}, \dot{\varphi}$, Equation (19) is obtained.

$$\begin{aligned} \dot{\psi} &= (\omega_z\cos\varphi + \omega_y\sin\varphi)/\cos\theta \\ \dot{\theta} &= \omega_y\cos\varphi - \omega_z\sin\varphi \\ \dot{\varphi} &= \omega_x + (\omega_z\cos\varphi + \omega_y\sin\varphi)\tan\theta \end{aligned} \tag{19}$$

Giving an initial value $\psi(0)$, $\theta(0)$, $\varphi(0)$ to this equation , one can use numerical integration to calculate the Euler angles $\psi(t)$, $\theta(t)$, $\varphi(t)$ at every time step. These Euler angles were then inputted into the transformation matrix, and the acceleration components a_x, a_y, a_z in the local coordinate system can be converted into a_X, a_Y, a_Z in the global coordinate system. Time-integrating these accelerations, it is thus possible to obtain the velocities v_X, v_Y, v_Z and displacements X, Y, Z in the global coordinate system.

2.2 Transformation into the vehicle coordinate system

Whenever there is any rotation of the passenger compartment, it is necessary to consider the displacement and orientation of the passenger compartment in order to obtain the dummy's displacement with respect to the passenger compartment's coordinate system O_0-$x'y'z'$. The vehicle orientation can be also obtained by using Euler angles and applying the same procedure as what was mentioned earlier. By rotating a coordinate system which has the same orientation as the global coordinate system in the sequence starting with yaw $\varPsi$, pitch $\varTheta$ and then roll $\varPhi$, the transformation equation between O_0-$x'y'z'$ and O-XYZ can be obtained.

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{bmatrix} \cos\varTheta\cos\varPsi & \sin\varPhi\sin\varTheta\cos\varPsi - \cos\varPhi\sin\varPsi & \cos\varPhi\sin\varTheta\cos\varPsi + \sin\varPhi\sin\varPsi \\ \cos\varTheta\sin\varPsi & \sin\varPhi\sin\varTheta\sin\varPsi + \cos\varPhi\cos\varPsi & \cos\varPhi\sin\varTheta\sin\varPsi - \sin\varPhi\cos\varPsi \\ -\sin\varTheta & \sin\varPhi\cos\varTheta & \cos\varPhi\cos\varTheta \end{bmatrix} \begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} \tag{20}$$

The acceleration components $a_{x'}$, $a_{y'}$, $a_{z'}$ and the angular velocity components p, q, r of the passenger compartment in its local coordinate system was obtained from the accelerometer attached onto the origin of vehicle coordinate system O_0 (Figure 1). Just like in Equation (19), the Euler angles $\varPsi(t)$, $\varTheta(t)$, $\varPhi(t)$ which represents the vehicle orientation can be obtained using Equation (21).

$$\begin{aligned} \dot{\varPsi} &= (r\cos\varPhi + q\sin\varPhi)/\cos\varTheta \\ \dot{\varTheta} &= q\cos\varPhi - r\sin\varPhi \\ \dot{\varPhi} &= p + (r\cos\varPhi + q\sin\varPhi)\tan\varTheta \end{aligned} \tag{21}$$

Substituting $\varPsi(t)$, $\varTheta(t)$, $\varPhi(t)$ into Equation (20), the displacement of the origin of the vehicle compartment can be obtained.

The position vectors of the point P on the dummy as well as the vehicle compartment origin O_0 are defined as $\boldsymbol{r}$, $\boldsymbol{r}_0$ respectively.

$$\boldsymbol{r}' = \boldsymbol{r} - \boldsymbol{r}_0 = (X - X_0)\boldsymbol{e}_X + (Y - Y_0)\boldsymbol{e}_Y + (Z - Z_0)\boldsymbol{e}_Z \tag{22}$$

The position vector of point P with respect to O0 is shown in Equation (22), and Equation (23) shows the components (x′, y′, z′) of this vector represented by the unit vectors ex′, ey′, ez′ of the vehicle coordinate system.

$$\begin{aligned} \begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} &= \begin{bmatrix} \boldsymbol{e}_{x'}\cdot\boldsymbol{e}_X & \boldsymbol{e}_{x'}\cdot\boldsymbol{e}_Y & \boldsymbol{e}_{x'}\cdot\boldsymbol{e}_Z \\ \boldsymbol{e}_{y'}\cdot\boldsymbol{e}_X & \boldsymbol{e}_{y'}\cdot\boldsymbol{e}_Y & \boldsymbol{e}_{y'}\cdot\boldsymbol{e}_Z \\ \boldsymbol{e}_{z'}\cdot\boldsymbol{e}_X & \boldsymbol{e}_{z'}\cdot\boldsymbol{e}_Y & \boldsymbol{e}_{z'}\cdot\boldsymbol{e}_Z \end{bmatrix} \begin{Bmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{Bmatrix} \\ &= \begin{bmatrix} \cos\varTheta\cos\varPsi & \cos\varTheta\sin\varPsi & -\sin\varTheta \\ \sin\varPhi\sin\varTheta\cos\varPsi - \cos\varPhi\sin\varPsi & \sin\varPhi\sin\varTheta\sin\psi + \cos\varPhi\cos\varPsi & \sin\varPhi\cos\varTheta \\ \cos\varPhi\sin\varTheta\cos\varPsi + \sin\varPhi\sin\varPsi & \cos\varPhi\sin\varTheta\sin\varPsi - \sin\varPhi\cos\varPsi & \cos\varPhi\cos\varTheta \end{bmatrix} \begin{Bmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{Bmatrix} \end{aligned} \tag{23}$$

(x', y', z') represents the components of dummy with respect to the vehicle compartment. Substituting X, Y, Z with the dummy's head displacement, X_0, Y_0, Z_0 with the vehicle displacement, and putting the coordinate system transformation matrix using Euler angles of the passenger compartment into the equation, , it is possible to obtain the displacement components (x', y', z') of the dummy's head with respect to the vehicle compartment.

3 Verification using simulation

3.1 Verification of the rear seat passenger model

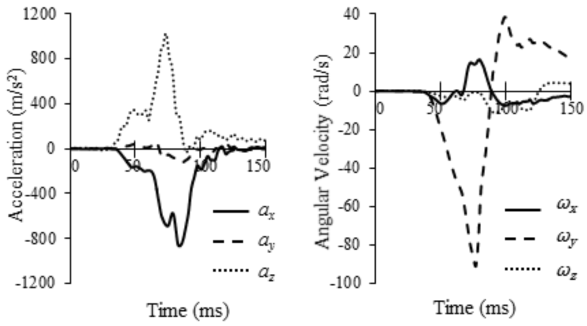
The method to calculate the head trajectory was verified through Finite Element Analysis (FEA). The model in the FEA simulation consists of the rear seat and the floor. The dummy was placed in a seating position in the rear seat wearing a three-point seatbelt. Then the deceleration of the vehicle was inputted into the floor of the vehicle.

The value used for the deceleration of the vehicle was based on a full frontal impact test of a Kei car with a large pitching rotation. In the test, the impact velocity was 55.6 km/h, and the Hybrid III AM50th was positioned in the rear seat wearing the three-point seatbelt. The vehicle deceleration curve was obtained from the accelerometer attached on the side sill beside the front passenger's seat.

Figure 5 shows the dummy's head acceleration and angular velocity obtained from the head accelerometer. The head accelerations (a_x, a_y, a_z) were represented in the head's local coordinate system.

Substituting the acceleration and angular velocity of the head COG into Equation (19) and using numerical integration, the Euler angles at a time t can be obtained. The modified Euler method was used for the numerical integration.

$$y_{i+1} = y_i + h \cdot f\left(x_i + h/2,\ y_i + (h/2) \cdot f(x_i, y_i)\right) \quad (i = 0,1,2,\ldots) \tag{24}$$

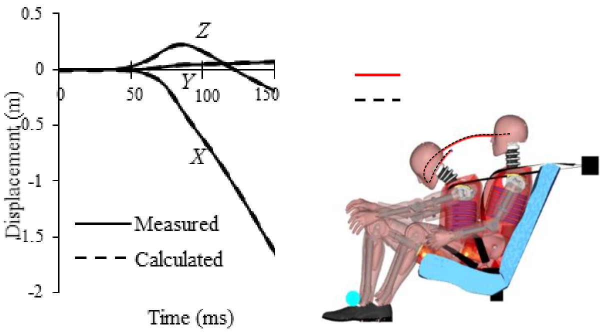


(a) Head acceleration (b) Head angular velocity

Figure 5. Data obtained from the accelerometer located in the CG of the rear seat passenger's head

Substituting these Euler angles into Equation (11), it is possible to obtain the transformation matrix [A] between the dummy's head coordinate system O1-xyz and the global coordinate system O-XYZ. By using [A], the head acceleration in the local coordinate system can be transformed in the global coordinate system. Then, by time-integrating the acceleration in the global coordinate system, the head velocity and trajectory can be obtained.

Figure 6(a) shows the comparison of the head displacement obtained from the FEA and the calculated result. The calculated trajectory of the head corresponds well with the FEA result.

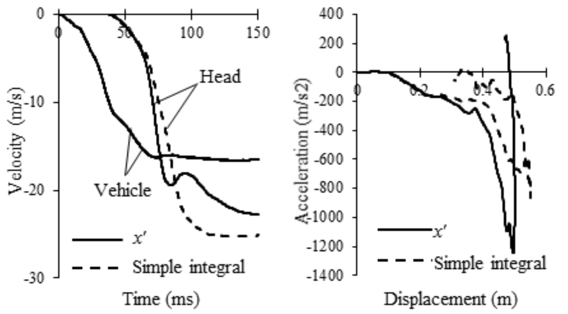


(a) Displacement (b) Head trajectory

Figure 6. Comparison of actual head trajectory and head trajectory calculated from data from the accelerometer in the rear seat passenger's head

The next step is to obtain the head trajectory with respect to the passenger compartment. The passenger compartment was regarded as a rigid body, the vehicle acceleration at the origin O_0 of the passenger compartment and the vehicle pitch angle were used as the input data. Following same procedures as mentioned in section 2.2, it is possible to obtain the displacement of O_0 in the global coordinate system and hence, the dummy's head trajectory with respect to the passenger compartment. Figure 6(b) shows that the calculated result corresponds well with the FEA result.

The forward component (x) from an accelerometer attached in a dummy's head is often integrated directly to obtain a Velocity-Time curve and an Acceleration-Displacement curve. However, it is not possible to obtain these curves correctly by integrating the dummy's head acceleration directly because the direction of the head coordinate system is always rotating with time. Therefore, by using the transformation matrix of Equation (23) and projecting the dummy's head acceleration onto O_0-$x'y'z'$, it is now possible to obtain the correct acceleration, velocity and displacement in the vehicle frontal direction (x').



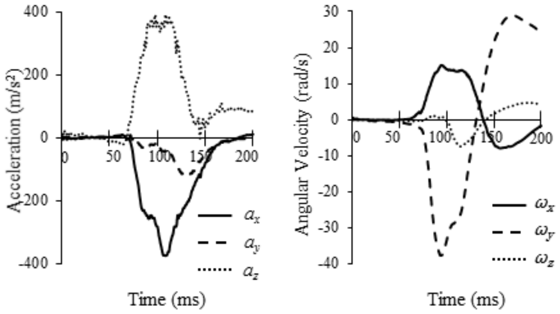
(a) Velocity (b) Acceleration-displacement

Figure 7. Velocity-time and acceleration-displacement history of the dummy's head from calculation (including dummy rotation) compared to these from simple linear acceleration integral (rear seat)

Figure 7 shows the Velocity-Time curve and the Acceleration-Displacement curve of the dummy's head and the vehicle (sled) in the x' direction. Displacement in this case means the dummy's displacement with respect to the passenger compartment. The result of integrating the head acceleration and vehicle deceleration directly without considering the rotation of the coordinate system is also shown in Figure 7. Due to the head rotation about the y-axis, there are obvious differences between the calculated result and the FEA result. Therefore, it is necessary to consider the rotation of the coordinate system in order to correctly estimate the energy characteristic and the time between the anterior loading and unloading on the dummy's head.

3.2 Verification of the driver model

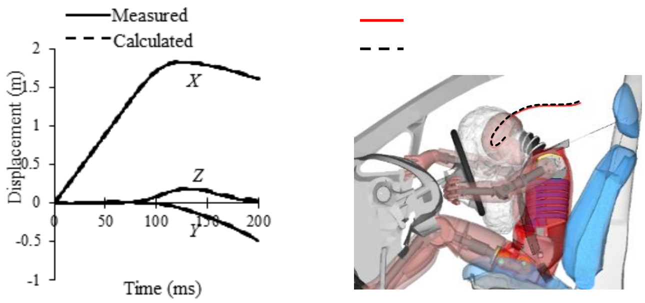
To verify the method in three-dimensional motion, a crash simulation of a driver undergoing rotation was conducted. The data from a small overlap frontal impact test of a small vehicle was chosen. This data was from IIHS (Insurance Institute for Highway). The impact velocity was 64.4 km/h and the lap ratio was 25%.The data from the accelerometer attached onto the floor of the passenger compartment during the crash test was used as the acceleration of the passenger compartment. This data, together with the angular displacement of the passenger compartment obtained from the video analysis of the crash test, were inputted into the model as the movement of the passenger compartment. The model consists of the driver seat, instrument panel, steering components and the vehicle compartment around the driver. The Hybrid III AM50th model was placed in a seating position on the driver's seat. Figure 8 shows the acceleration and the angular velocity of the dummy's head obtained from the accelerometer.



(a) Head acceleration (b) Head angular velocity

Figure 8. Data obtained from the accelerometer located in the CG of the dummy's head

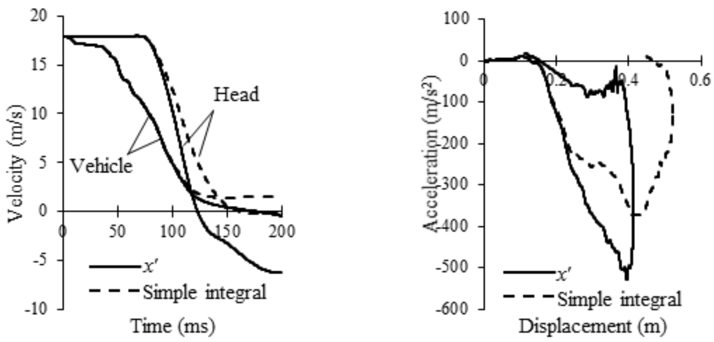
Following the procedures mentioned in the second chapter of this paper, the dummy's head trajectory with respect to the global coordinate system and the passenger compartment was obtained. Figure 9(a) shows the calculated result and the FEA result of the dummy's head trajectory in the global coordinate system. The calculated result corresponds well with the FEA result. Figure 9(b) shows the head motion trajectory with respect to the passenger compartment.



(a) Displacement (b) Head trajectory

Figure 9. Comparison of actual and calculated head trajectory calculated from data from the accelerometer in the driver's head

Same as what was done previously in 3.1 section, Figure 10 shows the Velocity-Time curve and the Acceleration-Displacement curve in the vehicle frontal direction (x′) with and without considering the rotation of the coordinate system. As clearly seen, there is the big difference between the two curves.



(a) Velocity (b) Acceleration-displacement

Figure 10. Velocity-time and acceleration-displacement history of the dummy's head from calculation (including dummy rotation) compared to simple linear integral of acceleration (driver dummy in small overlap test)

4 Verification of the sled test

4.1 Trajectory of target mark attached on dummy's head

In a car crash test, target marks are attached onto the dummy in order to obtain the trajectories of those points by tracing these marks in video analyses. However, since the accelerometer and the angular velocity sensor of the dummy's head are attached inside the head, it is not possible to obtain the trajectory of the sensor directly from the video analysis. Furthermore, the trajectory obtained by the method mentioned previously in this paper is the trajectory of the sensor. Therefore, calculating the trajectory of the target mark is necessary in order to compare the calculated result and the result from the video analysis.

To represent the trajectory of the target mark, the position vector rT1 in the global coordinate system, represented by the vector from the origin of the head coordinate system O1 to the target mark on the head, is necessary (Figure 11).

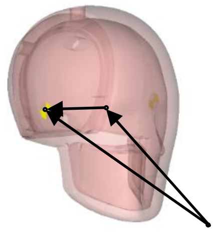


Figure 11. The definition of the position vectors

The position vector from O1 to the target mark with respect to the local coordinate system of the dummy's head is already known because the position of the target mark with respect to O1 is always constant. Therefore, the position vector rT1 can be obtained by transforming that vector from the local coordinate system to the global coordinate system using Equation (11). Hence, the position vector of the target mark rT is shown in Equation (25).

$$\boldsymbol{r}_T = \boldsymbol{r} + \boldsymbol{r}_{T1} \tag{25}$$

4.2 Verification of the sled test

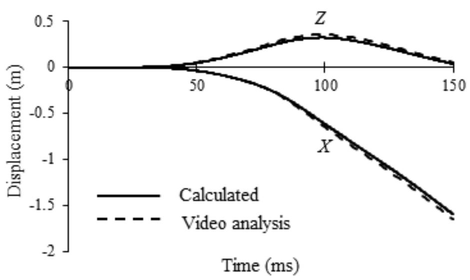


Figure 12. Comparison of the calculated result and the result from video analysis of sled test

In order to demonstrate real-world usability of this method, the dummy's head trajectory is calculated using the linear acceleration and angular velocity of the dummy's head obtained from the sled test. This calculated trajectory is then compared to the actual trajectory obtained from the video analysis. Figure 12 shows the comparison of the head displacement (X, Z) between the calculated result and the result from the video analysis. The trajectory obtained from the video analysis is only represented in two dimensions (X, Z) because of the limitations of the video recordings. It can be seen that the calculated result corresponds well with the result from the video analysis.

5 Modeling using LS-DYNA and MADYMO

The possibility of obtaining the head trajectory in the global coordinate system from the head and vehicle's acceleration and angular velocity in their respective local coordinate systems using LS-DYNA and MADYMO was also investigated. A head model with a local coordinate system was created in LS-DYNA and MADYMO. The acceleration and angular velocity of the dummy's head (Figure 8) used in section 3.2 were re-inputted into the head model in both software, and the trajectories of the head models in the global coordinate system were obtained. The head trajectories were then compared to the head trajectory in the original FEA.

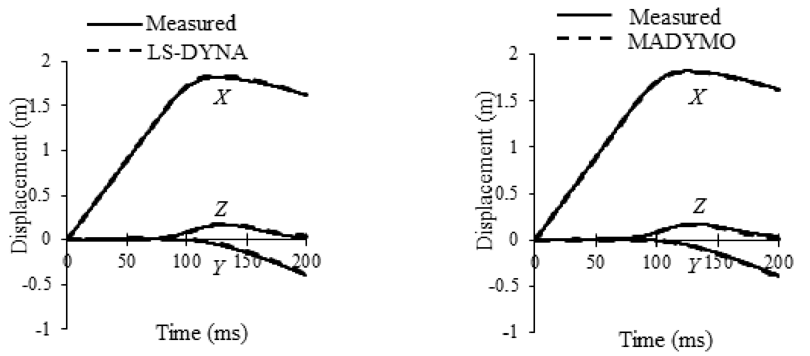
In LS-DYNA, the local coordinate system is defined by the keyword "DEFINE_COORDINATE_NODES", and the acceleration and angular velocity values are inputted using the keyword "BOUNDARY_

PRESCRIBED_MOTION_RIGID_LOCAL". Figure 13(a) shows the comparison between the original result and the result from the head model in LS-DYNA. Both curves were almost coincident to each other.

In MADYMO, the local coordinate system is defined by the keyword "JOIINT.FREE_ROT_DISP", and the acceleration is inputted using the keyword "MOTION.JOINT_ACC". However in this case, instead of an angular velocity, the angular acceleration needs to be inputted, also in "MOTION.JOINT_ACC". Therefore, the angular velocity is first differentiated to get the angular acceleration, which was then used in the "MOTION.JOINT_ACC" keyword. The following equation shows that even if the coordinate system rotates, the time-differential of an angular velocity about its axis will give the angular acceleration.

$$\begin{aligned}\frac{d\,\omega}{dt} &= \frac{d\omega_x}{dt}\,\mathrm{e}_x + \frac{d\omega_y}{dt}\,\mathrm{e}_y + \frac{d\omega_z}{dt}\,\mathrm{e}_z + \boldsymbol{\omega} \times (\omega_x\,\mathrm{e}_x + \omega_y\,\mathrm{e}_y + \omega_z\,\mathrm{e}_z) \\ &= \frac{d\omega_x}{dt}\,\mathrm{e}_x + \frac{d\omega_y}{dt}\,\mathrm{e}_y + \frac{d\omega_z}{dt}\,\mathrm{e}_z\end{aligned} \tag{26}$$

Figure 13(b) shows the comparison of the head displacements between the original result and the result from the head model in MADYMO. Both curves were also almost coincident with each other.



(a) LS-DYNA (b) MADYMO

Figure 13. Head displacement in original crash simulation compared to LS-DYNA and MADYMO simulations using the head's acceleration and angular velocity

6 Conclusion

In this research, the method to obtain the three-dimensional motion trajectory of a dummy's head with respect to the vehicle and the global coordinate system was proposed, and simulations and a sled test were conducted to verify this method. The possibility of obtaining the head trajectory using LS-DYNA and MADYMO was also investigated.

1. The method to obtain the three-dimensional motion trajectory of a dummy's head was demonstrated using the dummy's head and vehicle's acceleration and angular velocity in their respective local coordinate systems. Thus, the Velocity-Time curve and the Acceleration-Displacement curve of the dummy's head which considers the rotation of the coordinate system can be obtained.
2. The above-mentioned method was verified through FEA of the rear seat passenger and the driver undergoing a crash test. The calculated result corresponds well with the FEA result.
3. Using the sled test, the actual trajectory of the dummy's head from the video analysis was compared to the calculated trajectory. Both results were almost coincident to each other.
4. Using LS-DYNA and MADYMO, it is possible to obtain the trajectory by inputting the linear acceleration and the angular velocity into the local coordinate system of the head.

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