

# **The application and control optimization of electronic park brake in the park assist**

**Man XU, Yuke ZHEN, Kegang ZHAO**

*Guangdong Key Lab of Automotive Engineering, South China University of Technology, Guangzhou, 510640*

*Email: 1045811829@qq.com*

**Abstract:** In order to improve vehicle active safety, friction work of brake and jerk intensity are selected as the indexes to evaluate the brake process quality, the dynamic characteristics of transmission system composed by engine, hydraulic torque converter and drive shaft, are analyzed in the parking process. Based on the calculus of variations, by taking the rate of brake torque as the control variable, the weighted sum of the friction work and the square of control variables is served as the objective function which is proposed for the optimal process control. Through the solution of the optimization function with boundary constraints, the optimized vehicle speed curve used as the reference speed is obtained, the deviation between the reference speed and the actual speed acquired by wheel speed sensor is fed back to EPB (Electrical Park Brake), forming closed loop control. The simulation result on the platform (MATLAB /Simulink) exhibits that the vehicle securely parking within a certain time and distance. By means of the measuring the related parameters of the test car, the optimal speed curves of parking course are received. The test result verifies the feasibility of the control strategy.

**Keywords:** Park Assist; Electrical Parking System; Calculus of Variations

## **1 Introduction**

As an application of electronic technology in vehicle, the information of parking environment and vehicle was acquired and processed through the data acquisition system in the early stage of the development of park assist, then the system gave the driver some information for tips, which helped the driver control the car to complete the parking.

Based on the modern control theory, the technology of mechatronics is used in electronic park brake, this application upgrades the comfortable of operation. Nowadays, the EPB is widely used by all international automobile manufacturers.

As the security issues become more and more prominent, the speed control of park assist is particularly important, the important security issues of park assist are proposed in this paper. And a study which introduces EPB into park assist is carried out. By using EPB, the vehicle speed in parking process is well controlled and the active safety of parking is improved<sup>[1]</sup>.

## **2 Working principle of park assist for electronic park brake**

In parking process, cars will encounter not only static obstacles, but also dynamic obstacles, such as pedestrians, cars and so on. The most important of them is the pedestrian, this is because people behavior is difficult to predict for drivers. They may appear in parking path suddenly, drivers are often difficult to make correct response in this extremely dangerous condition, moreover, people subjectively tend to more uncertain in an emergency, which is more easily lead to accidents.

Whenever parking environment changes, the EPB can guarantee the automobile maintain an essential safety distance to surrounding environment, especially to the people. This function is the purpose of park assist of EPB.

By using ultrasonic or visual system, the spatial distance between vehicle and obstacles can be calculated by EPB during parking<sup>[2]</sup>. For example, when people suddenly appears in parking path, this system can immediately implement precise control and exert brake force to wheels according to control strategy, which will ensure the safety of pedestrians to the best advantage. This function is also the basic starting point of park assist with EPB.

The structure of park assist is shown in Figure.1, including following parts:

(1)Sensor: It's mainly used to obtain internal and external information of the vehicle, such as engine speed, throttle open degree, steering wheel angle, road gradient, wheel speed, brake system and the space position, etc.

(2)Electronic Control Unit: It can be divided into Main ECU (Main Electronic Control Unit) and EPB ECU (Electronic Control Unit of Electronic Park Brake). The Main ECU is a system for coordinated control and comprehensive management of the whole vehicle's subsystems. While the EPB ECU is the control unit for

deceleration and braking control [3]. The Main ECU transfers information from sensor to EPB ECU, then EPB ECU processes the information and sends the control command to actuator and feeds the command information back to Main ECU, so that the ECU can realize the coordinate operation of vehicle's various subsystems.

(3)Actuator: It receives the control command and acts to control the car.

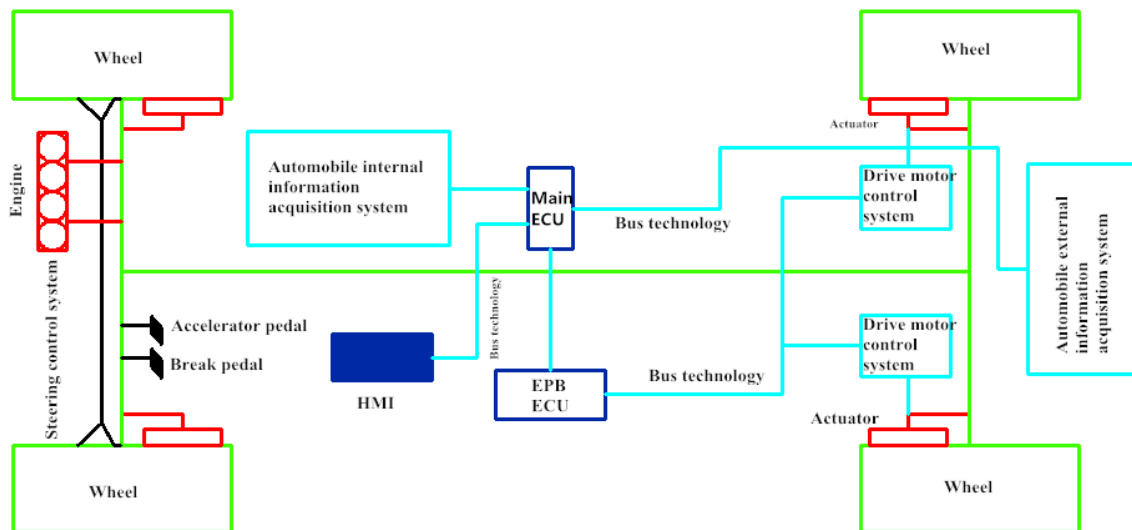


Figure.1 Structure diagram of park assist system with electronic parking brake system

In parking process, the system is possible to work in either of the following areas, namely environmental monitoring area and braked execution area. As shown in Figure.2, the threshold curve mapped of distance and speed is used as the boundary for operational region switch. When the car is braking at low speed and long distance condition, parking environment will be monitored by system in real time. Once the car is braking at high speed and close distance condition, the braking execution mechanism of the system will be operated immediately and control the vehicle speed to guarantee safety.

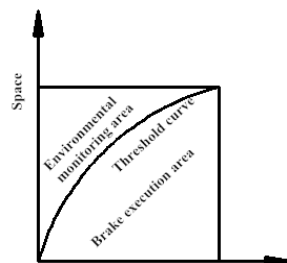


Figure.2 system working areas

Parking path can be planned when parking space is constructed completely. However, in track following stage of park assist, the vehicle speed is considered as a uniform speed in most of researches. In fact, pedestrians, vehicles and other factors in parking environment should be considered during parking. So the vehicle speed should change constantly under the control of EPB. First, the brake quality of parking brake is analyzed in this paper. Then the brake control strategy is worked out. Finally, the simulation of whole vehicle is carried out to verify the effectiveness of the control strategy.

### 3 Analysis on brake quality of parking brake

When the car into reverse, it is driven by the engine, while the brake is used to make the car finally stop. On one hand, the friction work generated by brake is directly related to the brake torque. On the other hand, the rate of brake torque is changed with time, which affects the jerk intensity of parking course. The friction work and jerk intensity are taken as the dynamic evaluation indexes of the braking<sup>[4]</sup>. Then the automobile power transmission system is analyzed. Finally, the influence factors of the brake quality are explored<sup>[5]</sup>.

The brake exerts torque on wheels during parking, torque affects the longitudinal acceleration of the vehicle changed with time. The longitudinal jerk intensity is defined as:

$$j = \frac{da}{dt} = \frac{r_w d^2 \omega_v}{dt^2} \quad (1)$$

Where  $j$  is jerk intensity,  $a$  is the longitudinal acceleration of the car,  $r_w$  is the rolling radius of the wheel and  $\omega_v$  is the angular velocity of the wheel.

If the jerk intensity is too large, it will cause the passenger uncomfortable. So it should be controlled in a reasonable range by subjective evaluation and calculation.

The speed is controlled by brake during parking, the energy generated by the friction between friction plate and brake disc will be dissipated in the form of heat. The friction work in parking process is:

$$W_B = \int_{t_o}^{t_f} T_B \omega_v dt \quad (2)$$

Where  $W_B$  is the slipping work,  $t_o$  is the parking starting time,  $t_f$  is the parking completion time and  $T_B$  is brake torque.

If the friction work is too large, it will reduce the braking efficiency, so it should be controlled in a certain range in parking process.

### 3.1 Analysis on the power transmission system of parking process

The output terminal of the engine is connected to the input shaft of hydraulic torque converter through relevant mechanism, and the output shaft of torque converter is connected to the input shaft of transmission. The transmission sends power to the wheels through final drive and differential.

Through the detailed analysis of parking powertrain system above, the influence factors of braking quality are explored. The formula established for engine is:

$$(I_e + I_p) \dot{\omega}_e = T_e - T_p - C_e \omega_e \quad (3)$$

Where  $I_e$  is the engine's rotational inertia,  $I_p$  is the rotational inertia of the torque converter pump wheel,  $\omega_e$  is the angular velocity of the engine,  $T_e$  is the output torque of the engine,  $T_p$  is the torque from the engine to the torque converter and  $C_e$  --the damping coefficient of the engine and the torque converter.

By analyzing vehicle power transmission, the torque that axle drives the wheel is:

$$\begin{aligned} T_{a\ le} &= (m \frac{dv}{dt} + mg \sin \alpha) r_w + J_w \frac{1}{r_w} \frac{dv}{dt} + fmg \cos \alpha r_w + F_w r_w \\ &= (mr_w^2 + J_w) \frac{d\omega_v}{dt} + (mg \sin \alpha + fmg \cos \alpha + F_w) r_w \end{aligned} \quad (4)$$

Where  $T_{a\ le}$  is the torque of the drive shift,  $m$  is the vehicle quality,  $v$  is the vehicle speed,  $g$  is the acceleration of gravity,  $\alpha$  is the gradient of the road,  $r_w$  is the wheel radius,  $J_w$  is the sum inertia moment of the wheel,  $f$  is the rolling resistance coefficient,  $F_w$  is the air resistance and  $\omega_v$  is the rotating angular velocity of the wheel.

The braking torque that the brake applies to the wheel is:

$$T_B = \mu p_B A_B r_B \quad (5)$$

Where  $\mu$  is the friction coefficient,  $p_B$  is the pressure of the hydraulic cylinder,  $A_B$  is the area of the hydraulic pressure and  $r_B$  is the effective radius of the brake friction plate.

By analyzing the torque converter, equation of motion is:

$$J_t \frac{d\omega_t}{dt} = J_t i_g i_0 \frac{d\omega_v}{dt} = T_t - \frac{T_{a\ le} + T_B}{i_g i_0} \quad (6)$$

Where  $T_t$  is the turbine output torque of the torque converter,  $i_g$  is the transmission ratio of the gearbox,  $i_0$  is the transmission ratio of the final drive,  $J_t$  is the inertia moment of the hydraulic torque converter output system,  $\omega_t$  is the turbine angular velocity of the hydraulic torque converter and  $T_B$  is the brake's braking torque.

Simultaneous equation (4) and equation (6):

$$J_t (i_g i_0)^2 \frac{d\omega_v}{dt} = T_t i_g i_0 - (mr_w^2 + J_w) \frac{d\omega_v}{dt} - T_B - (mg \sin \alpha + fmg \cos \alpha + F_w) r_w \quad (7)$$

From equation (7) can be obtained:

$$(mr_w^2 + J_w + J_t(i_g i_0)^2) \frac{d\omega_v}{dt} = T_t i_g i_0 - T_B - (mg \sin \alpha + fmg \cos \alpha + F_w) \quad (8)$$

Equation (8) can be written:

$$I_{equ} \dot{\omega}_v = T_t i_g i_0 - T_r - T_B \quad (9)$$

Where  $I_{equ} = mr_w^2 + J_w + J_t(i_g i_0)^2$ ,  $T_r = (mg \sin \alpha + fmg \cos \alpha + F_w)r_w$ .

In reverse parking process of automobile, the engine operates at constant speed. It uses constant speed PID feedback control strategy to ensure its speed constant.

The turbine output torque of torque converter is:

$$T_t = \lambda_p K \rho g D^5 n_p^2 \quad (10)$$

Where  $\lambda_p$  is the torque coefficient of the pump wheel,  $K$  is the torque ratio,  $\rho$  is the working fluid density of the hydraulic torque converter,  $g$  is the gravity acceleration,  $D$  is the working radius of the torque converter and  $n_p$  is the angular velocity of the pump wheel.

The gear ratio of the torque converter is:

$$i = \frac{n_T}{n_p} = \frac{\omega_v}{\omega_e} i_g i_0 \quad (11)$$

Torque coefficient  $\lambda_p$  and torque ratio  $K$  of pump wheel are the functions of gear ratio  $i$  of the torque converter.

But the engine speed is constant. The turbine output torque  $T_t$  of torque converter is a function of the angular velocity  $\omega_v$ , that is  $T_t = T_t(\omega_v)$ .

According to the least square method, the data acquired is linearized. The relationship is:

$$T_t = a_0 + a_1 \omega_v \quad (12)$$

Where  $a_0$  is a constant term and  $a_1$  is the monomial coefficient of fitting result.

Substitute equation (12) into equation (9):

$$\begin{aligned} I_{equ} \dot{\omega}_v &= T_t i_g i_0 - T_r - T_B \\ &= (a_0 + a_1 \omega_v) i_g i_0 - T_r - T_B \\ &= a_1 i_g i_0 \omega_v + (a_0 i_g i_0 - T_r) - T_B \end{aligned} \quad (13)$$

That is:

$$\dot{\omega}_v = \frac{a_1 i_g i_0}{I_{equ}} \omega_v + \frac{con_1}{I_{equ}} - \frac{T_B}{I_{equ}} \quad (14)$$

Where  $con_1 = a_0 i_g i_0 - T_r$ .

Substitute equation (14) into equation (1). The vehicle longitudinal jerk intensity in parking process can be obtained:

$$\begin{aligned} j = \frac{da}{dt} &= \frac{r_w d^2 \omega_v}{dt^2} = \frac{r_w a_1 i_g i_0}{I_{equ}} \dot{\omega}_v - \frac{r_w}{I_{equ}} \frac{dT_B}{dt} \\ &= \frac{r_w a_1 i_g i_0}{I_{equ}} \left( \frac{a_1 i_g i_0}{I_{equ}} \omega_v + \frac{con_1}{I_{equ}} - \frac{T_B}{I_{equ}} \right) - \frac{r_w}{I_{equ}} \frac{dT_B}{dt} \\ &= \frac{r_w (a_1 i_g i_0)^2}{I_{equ}^2} \omega_v + \frac{r_w a_1 i_g i_0 (con_1 - T_B)}{I_{equ}} - \frac{r_w}{I_{equ}} \frac{dT_B}{dt} \end{aligned} \quad (15)$$

Similarly, substitute equation (14) into equation (2), the friction work in parking process can be obtained:

$$W_B = \int_{t_0}^{t_f} T_B \omega_v dt = \int_{t_0}^{t_f} T_B(t) \omega_v(T_B, t) dt \quad (16)$$

From equation (15) and equation (16), the longitudinal jerk intensity and friction work both have the nonlinear relationship with braking torque and wheel angular velocity. And from equation (14), the wheel angular velocity and braking torque also has a nonlinear relationship. They work together to affect the braking quality<sup>[6]</sup>.

### 3.2 State space equation

Since the calculus of variations has been proposed from seventeenth century, it has penetrated into many fields, such as physics, economics, engineering and so on. By constructing the objective function, the unknown function which

lets the equation get the extreme value can be obtained [7]. The calculus of variations is chosen to optimize the braking quality in this paper. The state vector  $X$  is defined as:

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \omega_v \\ T_B \\ z \end{bmatrix} \quad (17)$$

Where  $\omega_v$  is the wheel angular velocity,  $T_B$  is the brake's braking torque and  $z$  is the ratio of the distance  $s(t)$  to  $r_w$  in parking process.

The control vector  $U$  is defined as:

$$U = [u] = \left[ \frac{dT_B}{dt} \right] \quad (18)$$

The dynamic equation of the brake is:

$$\begin{cases} \dot{x}_1 = \dot{\omega}_v = \dot{\omega}_v = \frac{a_1 i_g i_0}{I_{equ}} \omega_v + \frac{con_1}{I_{equ}} - \frac{T_B}{I_{equ}} = f_1(X, U, t) \\ \dot{x}_2 = \dot{T}_B = u = f_2(X, U, t) \\ \dot{x}_3 = x_1 = f_3(X, U, t) \end{cases} \quad (19)$$

Based on optimal control theory, the state space equation of the brake system is obtained:

$$\dot{X} = F(X, U, t) = \begin{bmatrix} f_1(X, U, t) \\ f_2(X, U, t) \\ f_3(X, U, t) \end{bmatrix} \quad (20)$$

#### 4 Brake control strategy

Using the calculus of variations to optimize the braking process, the weighted sum of the friction work and control variables square is structured as the objective function:

$$J = \int_{t_0}^{t_f} (k_w x_1 x_2 + u^2) dt \quad (21)$$

Where  $k_w$  is the weighted coefficient of the brake slipping work in the objective function.

The parameter that makes the  $J$  minimum is the objective value of optimization. The Lagrange Multiplier vector  $\lambda = [\lambda_1, \lambda_2, \lambda_3]^T$  is introduced and the generalized objective function is constructed:

$$J_d = \int_{t_0}^{t_f} [k_w x_1 x_2 + u^2 + \lambda^T (F - \dot{X})] dt \quad (22)$$

Then the Hamiltonian function is:

$$H = k_w x_1 x_2 + u^2 + \lambda_1 \left( \frac{a_1 i_g i_0}{I_{equ}} \omega_v + \frac{con_1}{I_{equ}} - \frac{T_B}{I_{equ}} \right) + \lambda_2 u + \lambda_3 x_1 \quad (23)$$

The canonical equations and the control equations are:

$$\begin{cases} \dot{X} = \frac{\partial H}{\partial \lambda} \\ \dot{\lambda} = -\frac{\partial H}{\partial X} \\ \frac{\partial H}{\partial u} = 0 \end{cases} \quad (24)$$

Simultaneous the equation (19) and equation (24):

$$\begin{cases} \dot{x}_1 = \frac{a_1 i_g i_0}{I_{equ}} \omega_v + \frac{con_1}{I_{equ}} - \frac{T_B}{I_{equ}} \\ \dot{x}_2 = u = -\frac{\lambda_2}{2} \\ \dot{x}_3 = x_1 \\ \dot{\lambda}_1 = -k_w x_2 - \frac{a_1 i_g i_0}{I_{equ}} x_1 \lambda_1 - \lambda_3 \\ \dot{\lambda}_2 = -k_w x_1 + \frac{\lambda_1}{I_{equ}} \\ \dot{\lambda}_3 = 0 \end{cases} \quad (25)$$

Start-stop constraint conditions of parking process:

$$\begin{cases} X(t_0) = [\omega_v(t_0) \ T_B(t_0) \ z(t_0)] = [\omega_{threshold} \ 0 \ 0] \\ X(t_f) = [\omega_v(t_f) \ T_B(t_f) \ z(t_f)] = [0 \ T_{Bf} \ dst / r_w] \end{cases} \quad (26)$$

Where  $\omega_{threshold}$  is the wheel speed at the beginning of the brake,  $T_{Bf}$  is the brake's braking torque at the end of the parking and  $dst$  is the actual journey of the car at the end of the parking.

If the pedestrians appear in parking path, the EPB can obtain the distance between vehicle and pedestrian by environment construction tool [8]. The distance between current position and parking space can also be obtained by the system. The parking distance is determined when the parking space is built. The car needs to stop in a certain period of time. The angular speed  $\omega_v$  changes from  $\omega_{threshold}$  to 0. The EPB control the braking torque changes from 0. All the parameters together constitute the constraint conditions of the equation (26), and the optimal s-t curve is got as the reference speed. The vehicle's speed in parking process is obtained by the wheel speed sensor, and the deviation between reference speed and wheel speed is put as the feedback information [9]. Based on this feedback, the EPB can control the brake and take both comfort and brake performance into account. More importantly, the active safety of parking is guaranteed.

## 5 Experimental verification

The total mass of the test car is 1510kg. The inherent resistance torque is  $T_r = 24.8NM$ . The transmission ratio from gearbox to vehicle transmission chain is 8.381. The relationship between the torque converter output and the angular velocity of wheel is  $T_t = -17.5\omega_v + 121$ . The braking distance is 2m.

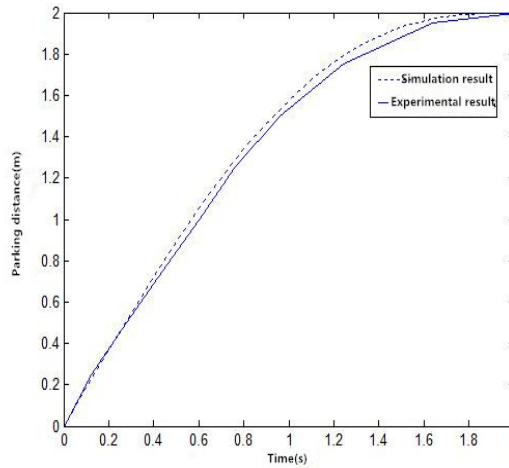


Figure3. Comparison of experimental results and simulation results of parking distance

Based on the above data, the speed optimized curve is generated. The experimental vehicle is parked according to the speed optimal curve, and its journey is compared with the target journey. As shown in Figure3. The comparison preliminarily shows that the brake control strategy proposed in this paper is feasible.

---

## 6 Conclusion

(1) In this paper, it is put forward that the friction work and jerk intensity in parking process are taken as the evaluation indexes of braking quality. By means of analyzing the major subsystems of engine, torque converter and motor drive shaft, the influence factors of quality of the brake are explored. Based on Calculus of Variations, the brake torque control strategy is studied, the dynamic equation of the brake is established. The simulation results show that the car can park safely in the expected way.

(2) Based on brake torque control strategy, the systemic solution of parking test is designed. Road resistance, speed ratio of gearbox and drive system are acquired through the experimental method of parking. A functional relation between output torque of torque converter and wheel angular velocity is fitted, then speed optimized curve is drawn. During parking, the vehicle follows the optimized curve. The test results show that the real parking distance curve and the target curve are consistent, and the feasibility of the control strategy is verified.

## Acknowledgement

This work was supported by the Fundamental Research Funds for Central Universities under Grant No.2154050.

## References

- [1] Skruch P. M., Dlugosz R., Kogut K., et al. The Simulation Strategy and Its Realization in the Development Process of Active Safety and Advanced Driver Assistance Systems[C]. SAE Technical Paper, 2015, 2015-01-1401
- [2] Ho G. J. Light-Stripe-Projection-Based Target Position Designation for Intelligent Parking-Assist System [J]. IEEE Transactions on Intelligent Transportation Systems, 2010, 11(4): 942-952
- [3] Yokoyama K., Iezawa M., Akashi Y., et al. Speed Control of Parking Assist System for Electrified Vehicle[C]. SAE Technical Paper, 2005, 2015-01-0316
- [4] Zhisheng Yu. Automobile theory (Fifth Edition) [M]. Beijing: China Machine Press, 2012: 89-98
- [5] Lee Y. O., Lee C. W., Chung H. B., et al. A Nonlinear Proportional Controller for Electric Parking Brake (EPB) Systems[C]. SAE Technical Paper, 2007, 2007-01-3657
- [6] Cheng K. P., Zhang Y., Chen H.. Planning and control for a fully-automatic parallel parking assist system in narrow parking spaces[C]. Intelligent Vehicles Symposium (IV), IEEE, 2013: 1440-1445
- [7] Peiju Ou. Variational method and its application [M]. Beijing: Higher Education Press, 2013: 90-114
- [8] Dingyu Xue and Yangquan Chen. Advanced Applied Mathematical Problem Solutions with MATLAB (Third Edition) [M]. Beijing: Tsinghua University Press, 2013: 276-280
- [9] Wang Bin, Guo Xuexun, Zhang Chengcai, Simulation and Experiment on Electrical Parking Brake System [J]. Journal of agricultural machinery 2013, 44(8): 45-49