Proceedings of the 11th International Forum of Automotive Traffic Safety, 2014, pp 444- 451
No. ATS.2014.507

Research on Validation Metrics for Multiple Dynamic Responses Comparison

Zhenfei ZHAN[1], Junqi YANG [1], Ling ZHENG[1], Yajing SHU[1], Yan FU [2], Ren-Jye YANG [2], Saeed Barbat [2]
[1]*College of Automotive Engineering, Chongqing University, Chong Qing, China, 400000*
[2] *Ford Research and Advanced Engineering, 2101 Village Rd, Dearborn, Michigan, USA, 48121*
Email:zhenfeizhan@cqu.edu.cn

Abstract:

Background:Increasing computer programs and models are developed to simulate vehicle crashworthiness, dynamic, and fuel efficiency. To maximize the effectiveness of these models, the validity and predictive capabilities of these models need to be assessed quantitatively.

Objective:This paper is aimed to develop objective validation metrics that has the desirable metric properties to quantify the discrepancy between multiple tests and simulation results in the form of time histories.

Method and Material: This study is carried out by utilizing principal components analysis (PCA) to address multivariate correlation. Then Area Metric is applied to calculate the disagreement between two data sets presented in the type of time histories to obtain the discrepancy of the overall distribution of data. Then, time histories are translated into frequency domain for the purpose of getting the features of errors in phase, topology and magnitude.

Results: The study results show thatthe values of measurements in magnitude, phase and topology are consistent with intuitive judgment based on graphics, which approves good performance of proposed metrics.

Conclusions:The proposed metrics satisfy the necessary properties of ideal metric in model validation and the investigated method is capable for automotive safety applications.

Keywords: Model validation, Multivariate dynamic responses, Principal component analysis, Frequency analysis

1 Introduction

With the ever shortening speed to market, computer aided engineering (CAE) has become a crucial tool for product development in automobile industry. Various computer programs and models are developed to simulate dynamic systems. Before using these models, their validity and predictive capabilities need to be assessed quantitatively. Model validation is the process of comparing CAE model outputs with test data in order to assess the validity or predictive capabilities of the CAE model for its intended usage. One of the critical tasks to achieve model validation is to select a validation metric that has the desirable metric properties for model assessment of a dynamic system with multiple functional responses[1, 2].

Developing quantitative model validation methods has attracted considerable researchers' interest in recent years[3]. They were applied to various model validation problems and showed significant potential. Statistical hypothesis testing is one approach to provide quantitative model validation. Fu et al. [4]and Zhan et al. [5]further exploited the Bayesian based validation methods for multivariate dynamic systems. However, these methods considered the whole distribution of interested time history, the agreements of the important features were not addressed. Error Assessment of Response Time Histories (EARTH) provides independent errors of phase, magnitude and slope but without an intuitive quantity to present the validity of a model. To tackle this problem, Zhan and Fu[6] investigated such method and proposed an enhanced version called Enhanced Error Assessment of Response Time Histories (EEARTH), which translates the three kinds of errors into a score between 0 and 100%. So it is convenience for engineers to conduct decision making.

However, those two methods for time histories conduct the process of model validation only in the time domain. Since frequency domain contains more intuitive information about phase, magnitude and shape, this paper investigates an innovative method for model validation of dynamic systems, which integrates both the analysis in time domain and frequency domain.

In this paper, a quantitative multivariate validation method and common process is proposed to quantitatively assess the agreements of important features of multiple dynamic responses simultaneously. It exploits several techniques such as principal component analysis (PCA)[7]and proposed validation method. The PCA approach is used to address multivariate correlation. To the validation metrics, in time domain, Area Metric is applied to calculate the disagreement between two data set presented in the type of time histories to obtain the discrepancy of the overall distribution of data. Then time histories are translated into frequency domain for the purpose of getting the features of errors in phase, topology and magnitude.

2 Multivariate Validation for Time Histories

2.1 PCA based dimension reduction

In multivariate data analysis, one major target to deal with highly correlated multivariate data is to remove the dependence amongst variables and reduce the dimension of variables. Amongst all techniques, PCA[8] is a well-established statistical method for dimensionality reduction and has been widely applied in data compression, image processing, exploratory data analysis, pattern recognition, and time series prediction. PCA involves a matrix analysis technique called eigenvalue decomposition. The eigenvalues and eigenvectors from decomposition represent the amounts of variation accounted for by each principal component and the weights for the original variables, respectively. Its main objective is to transform a set of correlated high dimensional variables to a set of uncorrelated lower dimensional variables (principal components). The important property of PCA is that the principal component projection minimizes the squared reconstruction error in dimensionality reduction. The PCA is not based on a probabilistic model, so no data distribution assumption is involved in a PCA transform process. In automotive safety applications, the output multiple responses are generated from the same crash event, hence, some of the responses are highly correlated. PCA could transform highly correlated original responses into uncorrelated lower dimensional responses. The judgment based on reduced principal components can be efficient and minimize the squared reconstruction error in dimension reduction.

To determine the proper number of principal components that should be retained is an important issue in PCA implementation. The first p eigenvalues are typically high, implying that most information is accounted for in the corresponding principal components. Thus, the intrinsic dimensionality p is obtained by calculating the cumulative percentage of the p eigenvalues (i.e., the total variability by the first p principal components) that is higher than a threshold value, say 95%, as is presented in expression 1. This implies that the retained p principal components account for 95% information of the original data.

$$\sum_{i=1}^{p}\lambda_i \Big/ \sum_{i=1}^{m}\lambda_i \geq 95\% \qquad (1)$$

2.2 Error assessments of dynamic responses

To the model validation of data represented in the form of time histories, in time domain the features of time curves, such as phase and frequency, cannot be obtained intuitively. So In this study, analysis in both time domain and frequency domain are associated to provide a comprehensive consideration of the magnitude error, phase error and shape error.

In the time domain, there exists a well-established metric called Area Metric. The Area Metric is firstly proposed by Ferson [3] to investigate the disagreements of repeated tests and simulations in the statistical features. In this study, the normalized and dimension-reduced data are utilized to construct the distribution of samples. They are represented as non-decreasing step function with a steady step. For the data are in the type of discrete points, the distribution of data in a time curve is expressed as

$$S_n(x) = \frac{\sum_{i=1}^{n} I(x_i, x)}{n} \qquad (2)$$

Where

$$\mathrm{I}(x_i, \mathrm{x}) = \begin{cases} 1 & x_i \leq x \\ 0 & x_i > x \end{cases}$$

It is implied that the $S_n(x)$ has the function of presenting the number of points below a specific quantity x. After plotting the $S_n(x)$ of two data sets that needed to be compared, the area between the two distributions can be calculated as the measure of discrepancy, as is denoted in Equation 3.

$$\mathrm{d}(S_n, S_n') = \int_{-\infty}^{\infty} |S_n(x) - S_n'(x)| dx \qquad (3)$$

For the analysis in frequency domain includes the error assessment of magnitude, if the normalized data are used to be transformed, the maximum and minimum quantities of two curves are all normalized into 1 and 0, which leads to losing the information of minimum and maximum values. Thus only test data for the analysis in frequency domain are under pretreatment of normalization while CAE data are just the dimension-reduced version with relative discrepancy to test data.

In this step, multiple methods aiming at transforming data into frequency domain exist, such as Laplace Transform (LT) and Fast Fourier Transform (FFT). In the process of transformation, original time histories are decomposed into plenty of signals of sine or cosine. By choosing the signals with big values in magnitude, the principal signals are determined.

Since the results in both time and frequency domain are obtained, metrics for errors of magnitude, phase and shape are introduced respectively. Analysis in frequency domain provides information of frequency and corresponding amplitude and phase, based on which the error in magnitude and phase for a certain response can be calculated with Equation 4 and Equation 5.

$$\varepsilon_{magnitude} = \sum_{i=1}^{q} \left(\frac{M_{Test\_i}}{\sum_{i=1}^{q} M_{Test\_i}} \left| \frac{M_{CAE\_i} - M_{Test\_i}}{M_{Test\_i}} \right| \times 100\% \right) \quad (4)$$

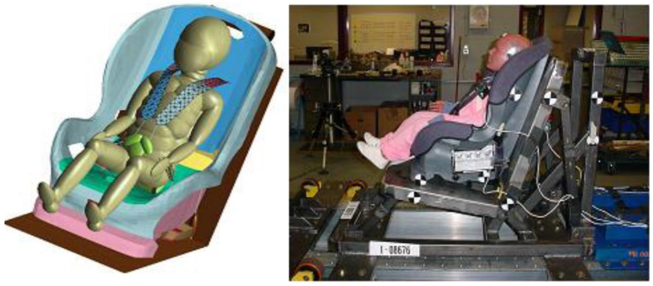
$$\varepsilon_{phase} = \sum_{i=1}^{q} \left(\frac{M_{Test_i}}{\sum_{i=1}^{q} M_{Test_i}} |P_{CAE_i} - P_{Test_i}| / 2\pi \times 100\% \right) \quad (5)$$

where q denotes number of frequency components, M_{CAE} and M_{Test} are the arrays of magnitudes while P_{CAE} and P_{Test} present phases for each frequency. With the computed ε_{phase}, error in shape can be obtained in Equation 6.

$$\varepsilon_{shape} = Area \times \varepsilon_{phase} \qquad (6)$$

3 Case Study

In this section, the proposed approach is demonstrated with a rear seat child restraint system with a hybrid III 3-year dummy model[9]. Figure 1 shows the CAE model and the rigid seat test set-up corresponding to the Evenflo Vanguard-5, respectively. Sixteen tests were conducted with different configurations, including different setting of seat cushion positions, top tether routing configurations and input crash pulses. Correspondingly, 16 sets of computer outputs were generated from the model.



(a) (b)

Figure 1 Rear seat child restraint system: (a) CAE model and (b) rigid seat test set-up

In the study, 8 responses (e.g., acceleration, deflection, force, etc.) in the form of histories are measured at a variety of locations of dummy and each contains 203 sampling points. Table 1 provides the meanings of those responses and Figure 2 plots the 8 responses and the simulation outputs in the same condition respectively. To demonstrate the proposed validation metrics, a pair of test and CAE data is selected.

Table 1. Reponses and corresponding descriptions

Variables	Description	Variables	Description
Response1	Head acceleration at x-direction	**Response5**	Chest acceleration at z-direction
Response2	Head acceleration at z-direction	**Response6**	Upper neck shear (Fx)
Response3	Pelvis resultant acceleration	**Response7**	Upper neck tension (Fz)
Response4	Chest acceleration at x-direction	**Response8**	Upper neck moment (My)

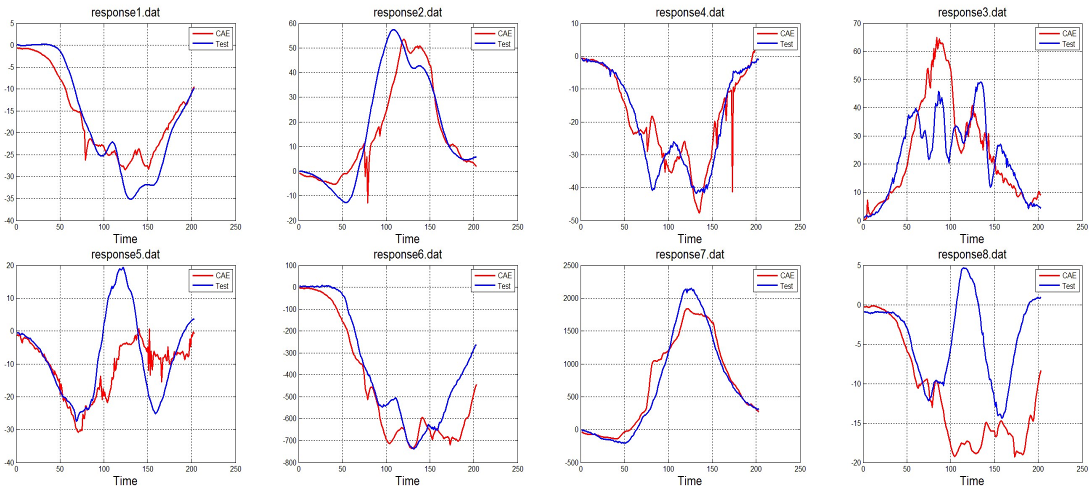


Figure 2. CAE and Test outputs for 8 responses

The original test matrix can be constructed in the size of. The technique of PCA is utilized to reduce the dimensions and address the correlation among responses. After the eigenvalues and corresponding eigenvectors are computed, principal components can be decided with eigenvalues. The eigenvalues are regarded as the measurements of the contributions of principal components. If 95% information of original data are required after projected on the new coordinates, the eigenvalues of chosen principal components should contribute at least 95% to the sum of eigenvalues. Figure 7 presents the contributions of each component to the whole.

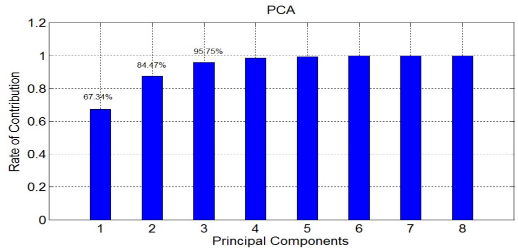
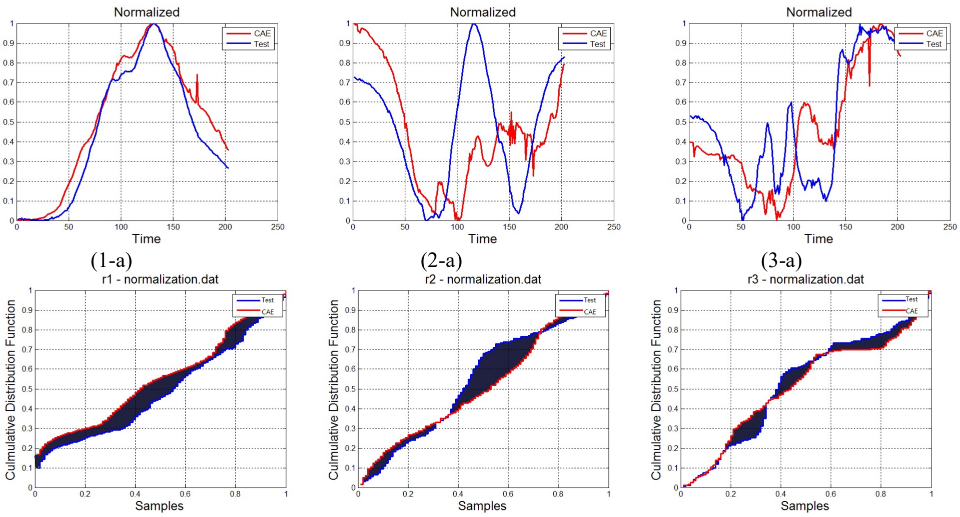


Figure 3. Rate of contribution for each principal component

Based on Figure 3, the first 3 components contain more than 95% information of the original data. Thus, the three corresponding eigenvectors are utilized as the updated projecting directions. After conducting the process of dimension reduction, 8 responses are transformed into test data matrix T* with 3 responses reserving information of data as much as possible. Similarly， outputs of CAE are projected onto the same axes to obtain the matrix C*.

After the process of pretreatment completed, model validation in time domain and frequency domain is conducted. Firstly, Area Metric is utilized to computing the degree of mismatch of CAE and Test. Secondly, data are transformed into frequency domain to reveal the features of phase and magnitude. Figure 4 presents the 'area' between CAE and Test in graphic way and quantitative way respectively.



(1-b) area=0.0639 (2-b) area=0.0643 (3-b)area=0.0423

Figure 4 Area Metric measurements for each principal component, graphically

The measurements of Area Metric offer the reference for the density distribution of samples in a certain response. Though the shape of curves can be recognized in some special conditions, but for complex waves the metric need to be combined with features in frequency domain. For instance, in Figure 9 (1-b), the CDF of Test is always lager than that of CAE, which presented as that the two curves never intersect with each other and are in similar tendency graphically. It is consistent with Figure 9 (1-a). Thus conclusion can be made that the CAE and Test outputs of principal response 1 share the similar shape. However, figures of response 2 and response 3 are with more complexity so that it is not easy and proper to conclude their similarity in topology. Therefore, further study are required in frequency domain.

Fast Fourier Transformation (FFT) is utilized to transform the time histories into frequency domain. The original curves are decomposed as multiple sine waves with the features of magnitude, frequency and phase. The transformed responses are depicted in Figure 5.

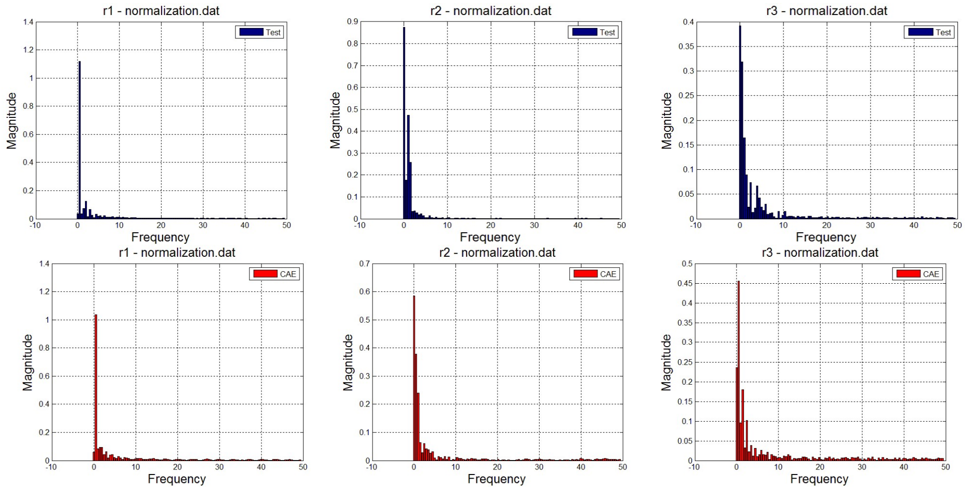


Figure 5. Frequency characteristics of principal components

100 frequency components are generated in each Test and CAE, but only the components with significant amplitude need to be taken into consideration for others effect trivially on the results. Computing based on dominate frequency is time saving and with high validity. Due to space limitations, only 3 frequency components are provided in Table 2 for each responses. The quantities are calculated according to the aforementioned Equation 4, Equation 5 and Equation 6.

Table 2. Error analysis in frequency domain

Test-r1		CAE-r1		Test-r2		CAE-r2		Test-r3		CAE-r3	
Magnitude	**Phase**	**Magnitude**	**Phase**	**Magnitude**	**Phase**	**Magnitude**	**Phase**	**Magnitude**	**Phase**	**Magnitude**	**Phase**
1.12	135.92	1.03	133.87	0.87	0	0.58	0	0.39	180	0.23	180
0.12	113.06	0.09	102.56	0.47	-42.13	0.24	-84.15	0.31	46.15	0.45	55.06
0.74	-11.31	0.09	67.764	0.25	98.44	0.063	-77.64	0.16	81.75	0.09	79.19
$\varepsilon_{magnitude}$	37.84%	ε_{phase}	2.99°	$\varepsilon_{magnitude}$	75.41%	ε_{phase}	-37.71°	$\varepsilon_{magnitude}$	70.89%	ε_{phase}	8.03°

As is presented in Table 2, total relative error in amplitude and mismatch in phase are computed based on the information in frequency domain. They act as the measurements of magnitude and phase shift directly. As is stated before, Area Metric provides reference of how samples distributed, and the phases of dominate frequencies reflect where the peak and valley appearance. Therefore, multiplying the quantities obtained in Area Metric in Figure 4 and Phase Mismatch in Table 2 offers the measurement for topology. Above all, the errors in magnitude, phase and shape are presented independently in Table 3.

Table 3. Analysis in frequency domain for sine curves

Error	Magnitude	Phase	Shape
r1	37.84%	2.99°	0.19
r2	75.41%	-37.71°	2.42
r3	70.89%	8.03°	0.34

The whole process of model validation based on the investigated method (metrics specifically) for the validation problem in engineering is completed. If the metrics really work, the quantitative measurements should be consistent with the intuitive judgments on original responses. So comparison is required between the quantities in Table 3 and the graphics in Figure 4. The curves of response 1 show that CAE and Test are consistent with each other in shape, magnitude and phase in a good condition graphically. And values of three types of errors in Table 3 reveal high validity of the simulation, which agrees with the curves in Figure 4. With respect to figures of r2 and r3, r2 has the largest discrepancy in the magnitude while CAE of r3 is in the same tendency of Test with fewer mismatches than r3. As can be seen in Figure 4, those characteristics are reflected in Magnitude and Shape in Table 3 intuitively too. Moreover, in terms of r2, the positions of peaks in Test and CAE differ a lot, which leads to that there exists a major error in phase shifting. So in summary, the investigated method for model validation is capable for such kind of problem.

Conclusions

In the problem of model validation, for the purpose of obtaining results successfully, it is of great importance to select or even develop proper metrics for specific applications. Appropriate metrics are required to be applicable on the deterministic system. In terms of dynamic system, the outputs of CAE and test are frequently in the form of time histories. However, it is not enough to analyze the discrepancy in just the values in time histories for such type of data. More features must be recognized to conduct overall analysis and make the decision whether to reject or accept the model, such as errors in magnitude, phase and topology. In this paper, methods for better representation of errors of magnitude, phase and topology are developed and investigated. Two main steps are included the proposed methodology. Firstly, a novel tool aiming at selecting principal components is proposed. Secondly, the proposed metrics are adopted to recognize the features in errors both in time domain and frequency domain. For the purpose of illustration, a practical engineering case of children restraint system is introduced and the whole process of validation is applied to it. The values of measurements in magnitude, phase and topology are consistent with intuitive judgment based on graphics, which approves good performance of proposed metrics.

However, the study in this paper only concentrate on how to present and quantify the errors in the three features. Therefore, to avoid obtaining a confusing result, the method to integrate several errors into a score quantity calls for further study.

References

[1] Oberkampf, W.L., Barone, M.F. *Measures of agreement between computation and experiment: validation metrics*. in Journal of Computational Physics. 2006. Changsha, China. 217(1):5-36. doi:10.1016/j.jcp.2006.03.037

[2] Oberkampf, W.L., Trucano, T.G. *Design of and Comparison with Verification and Validation Benchmarks*. Technical Report Sand No. 2006-5376C, Sandia National Laboratories, Albuquerque, New Mexico, USA.

[3] Ferson, S., Oberkampf, W.L., Ginzburg, L. *Model validation and predictive capability for the thermal challenge problem*. Computer Methods. In Applied Mechanics and Engineering, 2008. 197(29-32):2408-2430. doi:10.1016/ j.cma.2007.07.030

[4] Fu, Y., Zhan, Z., Yang, R.J., 2010. *A Study of Model Validation Method for Dynamic Systems*. In SAE World Congress, Detroit, USA, SAE 2010-01-0419.

[5] Zhan, Z., Fu, Y., Yang, R.J., Peng, Y. *An enhanced Bayesian based model validation method for dynamic systems*. In Journal of Mechanical Design, 2011a 133(4):041005. doi:10.1115/1.4003820.

[6] Zhan, Z., Fu, Y., Yang, R.J. *Enhanced Error Assessment of Response Time Histories (EEARTH) Metric and Calibration Process*. In SAE World Congress, Detroit, USA, 2011b. SAE 2011-01-0245.

[7] Joliffe, I.T. *Principal Component Analysis.* In *Springer*. 2002. New York, USA.

[8] Hotelling, H. *Analysis of a complex of statistical variables into principal components*. In *Journal of Educational Psychology*. 1933.24(6):417-441. doi:10.1037/h0070888.

[9] Zhan, Z., Fu, Y., Yang, R. J., Peng, Y., *Development and Application of a Reliability-Based Multivariate Model Validation Method*, International Journal of Vehicle Design, 60(3/4): 194-205, 2012.