Computational Robust Design Methods for Vehicle Crash Safety

GU Lei, XU Min, QI Guojun, GAO Xinhua, XU Liwei, XU Youzhong

Automobile Engineering Research Institute

CHERY Automobile Co., Ltd., WuHu, 241009

Contact E-mail：Gulei1@mychery.com, Xumin@mychery.com

ABSTRACT – The current push in vehicle structure safety design is focused on not only ensuring a desired rigidity, but also ensuring a desired energy absorption during crash. The balanced design between rigidity and energy absorption has been mainly originated in and been implemented in the product design stages by designer's experience and prototype test. More recently, however, it has been observed that much of a product performance and quality is determined by early design decisions with computer simulations. Computer simulation may take about 30% less design cycle time and may reduce about 40% design cost. In the first part of this paper, a typical Latin hypercube sampling is introduced and the robust design process based on the response surface method is discussed. Then, a flowchart of robust design optimization for crash safety is presented. Finally, some numerical examples are provided for demonstration.

Keywords:

1 Introduction

Continuous demands on cost reduction in product development have placed increasing emphasis on computer-aided engineering (CAE). Numerical methods such as finite element methods and numerical optimization have been widely served for this purpose. The application of multidisciplinary design optimization (MDO) to automotive vehicle structure design has been of significant interest over the last several years (Gu et al. 2001[1], Yang et al. 2002[2]). Kodiyalam et al. (2001) [3]reported a very significant reduction in computing time for such large-scale MDO problems – from months to days – through the efficient use of shared memory multiprocessor systems. Yang et al. (2002) [2]extended the work to robustness analysis for full vehicle crash safety.

In most applications, the optimization and robustness analysis are applied on surrogate models (or response surface models). After evaluated several response surface models for crashworthiness, Gu (2001)[4] recommended to use sequential replacement algorithm for polynomial based regression model and evaluated different order of polynomial models. Within a limited number of simulations, it was found that the second order polynomial response surface is good enough for many applications of crash safety. Another key element outlined in Gu et al. (2003)[5] is that it uses strategic sampling of the full-scale model, which is invoked only sparingly to save effort. The full-scale model is still used as the principal source of information about the system and therefore the computational cost per evaluation is not change. This strategy is effective and very suitable to parallel computing.

This paper presents some foundmental aspects of the safety optimization methodology and it's application.

2 Uniform Sampling

The method starts with useing strategic sampling of the full-scale model. The sampling method for computer analysis has been studied by many authors. Two major sampling technicals have been widely used for computer experimental design. McKay et al. (1979)[6] proposed a method of generating a set of experimental points $\mathbf{P}_n = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n\}$ called Latin hypercube sampling (LHS). LHS provides a more efficient estimate of the overall mean of the response than the estimate based on the simple random sampling or Monte Carlo sampling. Fang et al. (2000)[7], on the other hand, proposed the uniform design concept that allocates experimental point uniformly in the domain.

Uniform design seeks sampling points that are uniformly scattered on the domain. The fundamental idea in uniform design is that it uses L_p discrepancy as the criterion to measure the uniformity. The uniform design tries to find a set of samplings with maximizes the uniformity.

Suppose there are s factors of interest over a standard domain C^s. The goal here is to choose a set of n points $\mathbf{P}_n = \{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n\}$ such that these points are uniformly scattered in C^s. Let $M(\mathbf{P}_n)$ be a measure of the uniformity of $\mathbf{P}_n$; we seeks a set of $\mathbf{P}_n$ that maximizes the uniformity over all possible n points in C^s. Several uniform design critera can be used for this purpose as discussed in Fang et al.

The centered L_2 discrepancy criterion giving by Hickernell (1998)[8] was used by Gu et al. (2003)[5]:

$$M(P_n) = (\frac{13}{12})^s - \frac{2}{n}\sum_k^n \prod_l^s (1 + \frac{1}{2}|x_{kj} - 0.5|^2) - \frac{1}{2}|x_{kj} - 0.5|^2) + \frac{1}{n^2}\sum\sum\prod[1 + \frac{1}{2}|x_{kj} - 0.5|] -$$

$$- \frac{1}{n^2}\sum_k^n \sum_j^n \prod_i^s [\frac{1}{2}|x_{ji} - 0.5| + \frac{1}{2}|x_{ji} - x_{ki}|] \tag{1}$$

The L_2 discrepancy criterion proposed by Warnock can also be used for this purpose (1972)[9].

$$M(P_n) = 3^{-s} - \frac{2^{1-s}}{n}\sum_k^n \prod_l^s (1 - x_{kl}^2) + \frac{1}{n^2}\sum_k^n \sum_j^n \prod_i^s [1 - \max(x_{ki}, x_{ji})] \tag{2}$$

The global optimization algorithm, Threshold-accepting algorithm, is used to generate uniform samplings (Fang et al.).

The number of CAE simulations in uniform Latin hypercube sampling is determined by the total number of variables including control variables and noise variables in the optimization problem. To construct a reasonably accurate response surface, Gu et al. (2001)[4] proposed to use a minimum of 3N sampling points, where N being the total number of design variables. It is therefore recommended that the number of initial simulations should be between 3N – 4N for best results. Note that the number of levels for each variable is flexible.

3 Response Surface Models

Vehicle crash impact is a nonlinear event in terms of the structural and dummy responses. With increasing fidelity of the vehicle and dummy models, over a million degrees of freedom, a single crash analysis can require several hours of computing time on a state-of-the-art compute server with multiple processors. Even employing multiple processors with each crash simulation, the computational cost of these analyses along with the iterative nature of design optimization procedures prohibits rigorous optimization and robustness studies. In addition, crash analysis is unstable since some runs corresponding to design perturbations may fail due to modeling and element penetration issues. Hence it is critical that surrogate models (i.e. approximations) be constructed apriori using the results from a number of actual crash simulations for use with crashworthiness optimization. Two typical surrogate modeling methods, polynomial model and moving least square model, are considered in Yang et al. (2002)[2] for generating sufficiently accurate approximations of vehicle impact responses.

3.1 Polynominal Based Models

Polynomial-based regression models have been successfully used for vehicle safety analysis. Gu (2001)[4] proposed a sequential replacement algorithm in conjunction with Residual Sum of Squares (RSS) checking criterion to get the best fitting polynomial model.

The basic idea of the sequential replacement algorithm is that once two or more terms have been selected, it is determined that any of those terms can be replaced with another that gives a smaller RSS (Miller, 1990[10]). The procedure must converge as each replacement reduces the RSS that is bounded below.

Define the polynomial spaces G_l as:

$$G_0 = \{1\}$$

$$G_1 = G_0 \cup \{x_1, ..., x_n\}$$

$$G_2 = G_1 \cup \{x_1 x_1, ..., x_1 x_n, x_n x_n\}$$

where G_1 and G_2 are linear and quadratic space, respectively. n is the total number of design variables. In this study, design variables include gauges, material yield stress, vehicle impact speed and vehicle position.

The linear regression model $\hat{y}(\boldsymbol{x})$ for $y(\boldsymbol{x})$ is given by

$$\hat{y}(\boldsymbol{x}) = \sum_{i=1}^{p} a_i g_i(\boldsymbol{x}) \tag{3}$$

where $g_i(\boldsymbol{x}) \in G_l$ ($l = 1, 2$), p is the number of fitting terms in the fitted equation.

For linear model given by (3), define merit function:

$$J^2 = \sum_{i=1}^{m} [y_i - \sum_{i=1}^{p} a_i g_i(\boldsymbol{x})]^2$$

In the least square procedure, the best parameters a_i are picked to minimize J^2.

Many of the criteria that have been suggested in the literature to identify the best subset are monotone functions of RSS for subsets with the same number of independent variables (Myers, 1990[11]). Hence, the problem of finding the best polynomial can often be reduced to the problem of finding those polynomials of size p with minimum RSS. RSS is given by:

$$RSS = \sum_{i=1}^{l} (y_i - \hat{y}_i)^2 \tag{4}$$

where l is the total number of sampling points ($l > p$).

Most procedures that produce statistical inference depend heavily on the relation between the total and regression sums of squares. In the case of linear regression by linear polynomial, the relationship can be simplified as follows:

$$\sum_{i=1}^{l} (y_i - \bar{y})^2 = \sum_{i=1}^{l} (\hat{y}_i - \bar{y})^2 + \sum_{i=1}^{l} (y_i - \hat{y}_i)^2 \tag{5}$$

where $\bar{y}$ is the mean of y_i, and $\hat{y}_i = \hat{y}(\boldsymbol{x}_i)$. Equation 5 represents the following conceptual identity (Myers, 1990[11]):

(Total variability) = (Variability explained) + (Variability unexplained)

The coefficient of determination, often referred to, symbolically, as R^2 is related to (5) and a much used measure of the fit of the regression line. The R^2 is given by:

$$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2} \tag{6}$$

R^2 is surely a measure of the model's capability to fit the sampling data. The insertion of any new regressor into a model cannot bring about a decrease in R^2. Though there are rules and algorithms that allow for selection of best model, the statistic itself is not conceptually prediction oriented. It is not recommended as a sole criterion for choosing the best prediction model from a set of candidate models.

The adjusted R^2, denoted by $\overline{R}^2$, is used by many authors as a criterion for identifying the best prediction models in subset selection. The $\overline{R}^2$ can guard against overfitting by including marginally important model terms at the expense of error degrees of freedom. Adjusted R^2 is given by:

$$\overline{R}^2 = 1 - \frac{\Sigma (y_i - \hat{y}_i)^2}{\Sigma (y_i - \overline{y})^2} \times \frac{(l-1)}{(l-p)} \tag{7}$$

The procedure must converge as each replacement reduces the RSS that is bounded below. In practice, the procedure usually converges very rapidly. Sequential replacement algorithm normally used in conjunction with stepwise selection. It can be obtained by taking the stepwise selection and applying a replacement procedure after each new term is added. Results of sequential replacement are usually better than those from stepwise. Sequential replacement requires more computational time than that required by stepwise selection but it is feasible to apply to problems with several hundreds terms in the solution sets when subset of 20-30 terms are required. Another advantage for sequential replacement algorithm is that it does not have artificial parameter to be turned.

3.2 Moving Least Square Models

Moving least square (MLS) was originated in curve and data fitting in 1980s. The MLS approach has become quite popular since it has been successfully applied to construct shape functions in mesh free method (Belytschko et al. 1994[12]). Lancaster and Salkanskas (1980)[13] gave an excellent summary of MLS.

The MLS interpolate $u^{\boldsymbol{h}}(\boldsymbol{x})$ of the function $u(\boldsymbol{x})$ is defined in the domain □ by:

$$\mathrm{u}^h(x) = \sum_{i=1}^{p} \mathrm{P}_i(x)\alpha_i(x) = P^T(x)\alpha(x) \tag{8}$$

where p is the number of terms in the basis, $P_i(x)$ are monomial basis functions, and $\square_i(\boldsymbol{x})$ are their coefficients.

A linear basis is provided by

$$\boldsymbol{P}^{\boldsymbol{T}}(\boldsymbol{x}) = [1 \quad x] \text{ IN 1-D}$$

and a quadratic basis by:

$$\boldsymbol{P}^{\boldsymbol{T}}(\boldsymbol{x}) = [1 \quad x \quad x^2] \text{ IN 2-D}$$

The coefficients $\square_i(\boldsymbol{x})$ can be obtained at any point $\boldsymbol{x}$ by minimizing a weighted, discrete L_2 norm:

$$\boldsymbol{J} = \sum_{\boldsymbol{i}=1}^{\boldsymbol{p}} \boldsymbol{w}(\boldsymbol{x} - \boldsymbol{x_i})\left[\boldsymbol{P^T}(\boldsymbol{x_i})\alpha(\boldsymbol{x}) - \boldsymbol{u_i}\right]^2 \tag{9}$$

The stationary condition of $\boldsymbol{J}$ with respect to $\alpha(x)$ leads to:

$$\alpha(\boldsymbol{x}) = \boldsymbol{A}^{-1}(\boldsymbol{x})\boldsymbol{B}(\boldsymbol{x})\boldsymbol{u} \tag{10}$$

where

$$A(x) = \sum_{i=1}^{p} w(x - x_i)P(x_i)P(x_i)^T \tag{11}$$

$$\boldsymbol{B}(\boldsymbol{x})=\left[\boldsymbol{w}(\boldsymbol{x}-\boldsymbol{x}_1)\boldsymbol{P}(\boldsymbol{x}_1)\ \ .\ \ ..\ \ \boldsymbol{w}(\boldsymbol{x}-\boldsymbol{x}_n)\boldsymbol{P}(\boldsymbol{x}_n)\right] \tag{12}$$

where n is the number of points in the domain of influence. Hence we have:

$$u^{\boldsymbol{h}}(\boldsymbol{x})=\boldsymbol{P}^{\boldsymbol{T}}(\boldsymbol{x})\alpha(\boldsymbol{x})=\sum_{\boldsymbol{i}=1}^{\boldsymbol{p}}\boldsymbol{N}_{\boldsymbol{i}}(\boldsymbol{x})\boldsymbol{u}_{\boldsymbol{i}} \tag{13}$$

where the interpolate function $\boldsymbol{N}_i(\boldsymbol{x})$ is given by:

$$\boldsymbol{N}_{\boldsymbol{i}}(\boldsymbol{x})=\sum_{\boldsymbol{j}=1}^{\boldsymbol{p}}\boldsymbol{P}_{\boldsymbol{j}}(\boldsymbol{x})\left[\boldsymbol{A}^{-1}(\boldsymbol{x})\boldsymbol{B}(\boldsymbol{x})\right]_{\boldsymbol{ji}} \tag{14}$$

4 Reliability-Based Design Optimization (RBDO)

4.1 Design Optimization and RBDO

With the help of response surface method, the optimization problem can be described as:

$$\begin{aligned} &\text{Minimize} && f(\boldsymbol{x}) \\ &\text{Subject to} && g_{\boldsymbol{j}}(\boldsymbol{x})\le 0,\ j=1,2,\Lambda\ ,m \\ & && \boldsymbol{x}^{\boldsymbol{l}}\le\boldsymbol{x}\le\boldsymbol{x}^{\boldsymbol{u}} \end{aligned} \tag{15}$$

where m is the total number of constraints. Based on the surrogate models, the deterministic optimization was performed by using sequential quadratic programming (SQP) with mixed types of design variables. After obtaining a deterministic optimum, a robustness assessment is made by modeling the variables with uncertainties. These uncertainties result from many sources, e.g., gages, materials, test variability, etc. Due to the uncertainties, the design variables need to be treated as random variables. Robustness assessment is performed by constructing an empirical cumulative distribution function (CDF) of the response variable generated from Monte Carlo simulation.

The sequential replacement response surface can be used as a surrogate model to perform both the deterministic optimization and robustness assessment. Because the response surface model is represented by a small system of equations, it can be evaluated quickly at very little computational expense compared to the full finite element simulation. Thousands of samples may be required to realize a good representation of the response probability distribution when Monte Carlo simulation is used.

Deterministic optimization is commonly used for most vehicle design applications. The variability of design parameters is only considered through the use of safety factors. It is important to incorporate uncertainty in engineering design optimization and develop computational techniques that enable engineers to make efficient and reliable decisions. Methods available in the literature to this goal include reliability-based design optimization (Choi et al. 1996[14], Du and Chen, 2002[15]) and robust design (Gu et al. 2003[5]).

The reliability-based design optimization problem can be generally formulated as (Choi et al. 1996[14]):

$$\begin{aligned} &\text{Minimize} && f(\mu_{\boldsymbol{x}}) \\ &\text{s.t.} && R_j \ge P(g_j(\boldsymbol{x})\le 0),\ j=1,\dots,m \\ & && \mu_{\boldsymbol{x}}^{\boldsymbol{l}}\le\mu\le\mu_{\boldsymbol{x}}^{\boldsymbol{u}} \end{aligned} \tag{16}$$

where f and g_j are the objective and constraint functions, respectively, $\boldsymbol{x}$ is the random design vector with mean μ_x, m is the number of probabilistic constraints, R_j is the desired reliability and the probabilistic constraints are described by the performance function $g_j(\boldsymbol{x})$ with $g_j(\boldsymbol{x}) > 0$ indicates failure. It can be rewritten as:

$$\begin{aligned} &\text{Minimize} && f(\mu_{\boldsymbol{x}}) \\ &\text{s.t.} && P\left(g_j(\boldsymbol{x})\le 0\right)-\Phi\left(-\beta_{\boldsymbol{t}}\right)\le 0,\ j=1,\dots,m \end{aligned}$$

$$\mu_x^l \le \mu \le \mu_x^u \tag{17}$$

where Φ is the cumulative distribution function for standard normal distribution and β_t is the prescribed confidence level.

Probability of failure is statistically defined by a cumulative distribution function $F_{g_j}(0)$ as

$$P(g_j(\boldsymbol{x}) \le 0) = F_{g_j}(0) = \int_{g_j(x)\le 0} \Lambda \int f_{\boldsymbol{x}}(\boldsymbol{x}) d\boldsymbol{x} \le \Phi(-\beta_t)$$

where $f_x(\boldsymbol{x})$ is a joint probability density function, which needs to be integrated. Approximate probability integration methods, e.g., the first-order reliability method (FORM), have been widely used to provide efficient and adequately accurate solutions (Haldar and Mahadevan, 2000[12]).

Through inverse transformation, the probabilistic constraint in Eq. 17 can be further expressed in two distinct forms as:

Minimize $\quad f(\mu_x)$

s.t. $\quad \beta_{S_j} = \left(-\Phi^{-1}\left(F_{g_j}(0)\right)\right) \ge \beta_t$

$$\mu_x^l \le \mu \le \mu_x^u \tag{18}$$

or

Minimize $\quad f(\mu_x)$

s.t. $\quad g_{P_j} = F_{g_j}^{-1}(\Phi(-\beta_t)) \ge 0$

$$\mu_x^l \le \mu \le \mu_x^u \tag{19}$$

where β_{s_j} and g_{p_j} are the safety reliability index and the probabilistic performance measure for the i^{th} probabilistic constraint, respectively. The reliability index approach (RIA) uses the reliability index (Eq. 18) to describe the probabilistic constraint. Similarly, it is referred as the performance measure approach (PMA), if the probabilistic performance measure in Eq. 19 replaces the probabilistic constraint in Eq. 17.

In FORM, the first-order safety reliability index β_s is obtained using first order reliability methods. It is formulated as an optimization problem as shown in (17).

The optimum point on the failure surface is referred as the most probable failure point (MPP). Any MPP search algorithm specifically developed for FORM and general optimizer can be used to solve Eq. 17.

4.2 Hasofer-Lind Method

For normal random variables, the Hasofer-Lind (H-L) method is widely used to solve β in (17). In general, for random variables represented by vector $\boldsymbol{x} = (x_1, \ldots, x_n)$ in the original coordinate system and $\boldsymbol{x}' = (x_1', \ldots, x_n')$ in the reduced coordinate system, where $x_i' = \dfrac{x_i - \mu_{x_i}}{\sigma_{x_i}}$. The limit state $g(\boldsymbol{x}') = 0$ is a nonlinear function. $g(\boldsymbol{x}') < 0$ denotes the failure state. The H-L reliability index β_{HL} is defined as the minimum distance from the origin to the design point on the limit state in the reduced coordinates. The minimum distance point on the limit state surface is the most probable failure point. The MPP represents the worst combination of the stochastic variables and is appropriately named the design point or the most probable point of failure.

For nonlinear function $g(\boldsymbol{x}')$, the computation of the minimum distance becomes an optimization problem:

$$\text{Minimize} \quad \|\boldsymbol{D}\| = \sqrt{\boldsymbol{x}'^{T}\ \boldsymbol{x}'}$$
$$\text{S.T.} \quad g(\boldsymbol{x}') = 0 \tag{20}$$

where $\boldsymbol{x}'$ represents the coordinates of the checking point on the limit state equation.

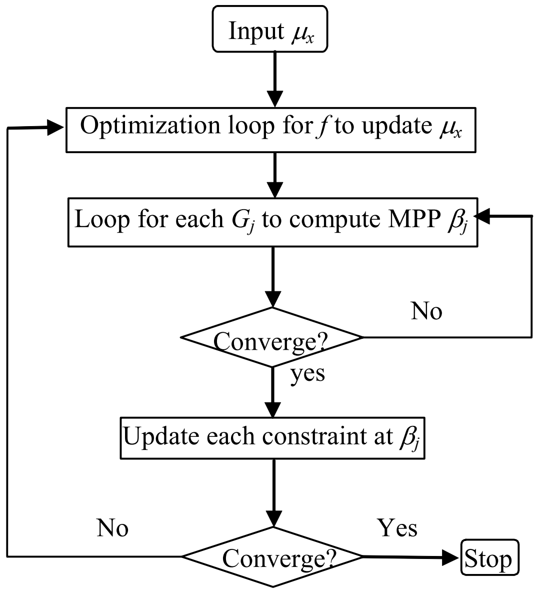

Fig.1 Flow chart of Hasofer-Lind Method

Using the method of Lagrange multipliers, we can obtain the minimum distance as (Haldar and Mahadevan, 2000[12])

$$\beta_{HL} = -\frac{\sum_i^n x_i'^{*} \left(\frac{\partial g}{\partial x_i'}\right)^{*}}{\sqrt{\sum_i^n \left(\frac{\partial g}{\partial x_i'}\right)^{2*}}} \tag{21}$$

where $\left(\frac{\partial g}{\partial x_i'}\right)^{*}$ is the i^{th} partial derivative evaluated at the design point with coordinates $(x_1'^{*},\ldots, x_n'^{*})$. The asterisk after the derivative indicates that it is evaluated at $\boldsymbol{x}'^{*}$. The design point in the reduced coordinates is given by:

$$x_i'^{*} = -\alpha_i \beta_{HL}$$

where

$$\alpha_i = \frac{\left(\frac{\partial g}{\partial x_i'}\right)^{*}}{\sqrt{\sum_i^n \left(\frac{\partial g}{\partial x_i'}\right)^{2*}}} \tag{22}$$

are the direction cosines along the coordinate axes x_i'. In the space of the original coordinates, we find the design point to be $x_i'^* = \mu_{x_i} - \alpha_i \sigma_{X_i} \beta_{HL}$.

The following algorithm has been used to computer β_{HL} and MPP $\boldsymbol{x}'^*$(Rackwitz, 1976[16])

Step 1. Define the appropriate limit state function.

Step 2. Assume initial values of the design point $\boldsymbol{x}^*$. Typically, the initial design point may be assumed to be at the mean values of the random variables. Calculate the reduced variants $x_i'^* = (x_i^* - \mu_{x_i}) / \sigma_{x_i}$.

Step 3. Calculate $\left(\frac{\partial g}{\partial x_i'}\right)^*$ and α_i at $x_i'^*$.

Step 4. Obtain the new design point $x_i'^*$.

Step 5. Substitute new $x_i'^*$ into $g(\boldsymbol{x}'^*)=0$ and solve for β_{HL}.

Step 6. Resolve $x_i'^*$ by using new β_{HL} from Step 5.

Step 7. Repeat step 3 through step 6 until β_{HL} convergences.

Fig. 1 shows the flow chart of the H-L method. It needs to be addressed the above Hasofer-Lind method is applicable only for normal variables. If not all the variables are normally distributed, it is necessary to transform the non-normal variables into equivalent normal variables. The flowchart also shows that H-L method is a double-loop method that involves an outer loop for optimization and *m* inner loops for MPPs of constraints. The double-loop method is computational expensive.

4.3 Single-Loop-Single-Vector Method

Chen et al. (1997 [17]) proposed an efficient method for probabilistic design optimization (17) which employs a single-loop-single-vector (SLSV) approach.

The method is derived in a reduced, uncorrelated, normalized parameter space is denoted by x', where, $x'_{\boldsymbol{i}} = \frac{x_{\boldsymbol{i}}}{\sigma_{x_{\boldsymbol{i}}}}$.

The mean value of x_i is denoted by μ_{xi}. The most probable point (MPP), $\boldsymbol{x'}^*$, determined by (20), leads to the following equation,

$$\left| \boldsymbol{x}'^* - \mu_{\boldsymbol{x}'} \right| = \alpha^* \beta \tag{23}$$

where α^* is a unit vector normal to the limit surface, and

$$\alpha_i^* = \frac{\sigma_{x_i} \left(\frac{\partial g}{\partial x_i} \right)^*}{\sqrt{\sum_i^n \sigma_{x_i}^2 \left(\frac{\partial g}{\partial x_i} \right)^{2*}}} \tag{24}$$

The key to the SLSV method is to recognize that $\boldsymbol{x}'^*$ and $\mu_{x'}$ are related by β, which is known provided that β is specified as the desired safety factor, β_0. With this insight, the optimization problem stated in (17) can be solved in the following procedure,

$$\begin{aligned} \text{Minimize:} \quad & f(\mu_x^{(k)}) \\ \text{s.t.} \quad & G_{\boldsymbol{j}}(\boldsymbol{x}'^{(\boldsymbol{k})*}) \geq 0 \\ & \mu_{\boldsymbol{x}}^{\boldsymbol{l}} \leq \mu \leq \mu_{\boldsymbol{x}}^{\boldsymbol{u}} \end{aligned} \tag{25}$$

where: $x'_i = \dfrac{x_i}{\sigma_{x_i}}$

$$x'^{*}_{j} = \mu_z^{(k)} - \beta_0 \alpha_j^{*(k-1)}$$

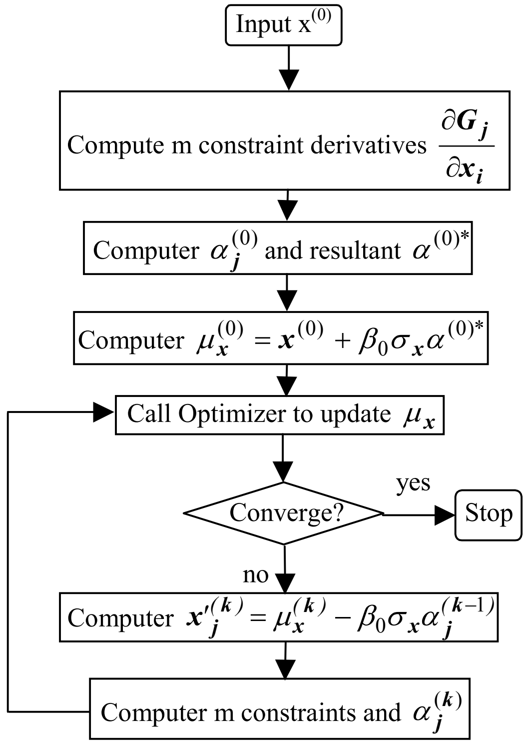

Fig. 2 Flowchart of SLSV Method

The SLSV method is efficient because there is no need to calculate a safety index, β_j, for each constraint. This eliminates the entire inner loops for the H-L method. The method is still computational expensive compared to the deterministic optimization since it does require evaluating the derivates at MPP for each constraint. A flow chart illustrating implementation of the method is shown in Figure 2.

4.4 Mean Value First Order Reliability Method (MVFORM)

The mean value first order method derives its name from the fact that it is based on a first-order Taylor series approximation liberalized at the mean values of the random variables. A Taylor series expansion of the function *Y=g(x)* about the mean value gives

$$Y = g(\mu_x) + \sum_i^n \frac{\partial g}{\partial x_i}(x_i - \mu_{x_i}) + \frac{1}{2}\sum_i \sum_j \frac{\partial^2 g}{\partial x_i \partial x_j}(x_i - \mu_{x_i})(x_j - \mu_{x_j}) + \dots \quad (26)$$

where the derivatives are evaluated at the mean values of the random variables ($x_1, x_2, \dots, x_n$), and the μ_{xi} is the mean value of x_i. The first order approximate mean and variance of Y are given by

$$\mu_Y \approx g(\mu_{x_1}, \mu_{x_2}, \dots, \mu_{x_n}) \quad (27)$$

$$\sigma_Y^2 \approx \sum_i^n \left(\frac{\partial g}{\partial x_i} \right)^2 \sigma_{x_i}^2 \tag{28}$$

The reliability index can be calculated by taking the ratio of the mean and the standard deviation of y,

$$\beta = \frac{\mu_{\boldsymbol{Y}}}{\sigma_{\boldsymbol{Y}}} \tag{29}$$

Finally, a completely deterministic statement of the design optimization problem with a deterministic objective function and probabilistic constraints is

$$\begin{aligned} &\text{Minimize} \quad && f(\mu_{\boldsymbol{x}}) \\ &\text{s.t.:} && \beta_{\boldsymbol{j}} \ge \beta_{0\boldsymbol{j}} \\ &\mu_{\boldsymbol{x}}^{\boldsymbol{l}} \le \mu \le \mu_{\boldsymbol{x}}^{\boldsymbol{u}} \end{aligned} \tag{30}$$

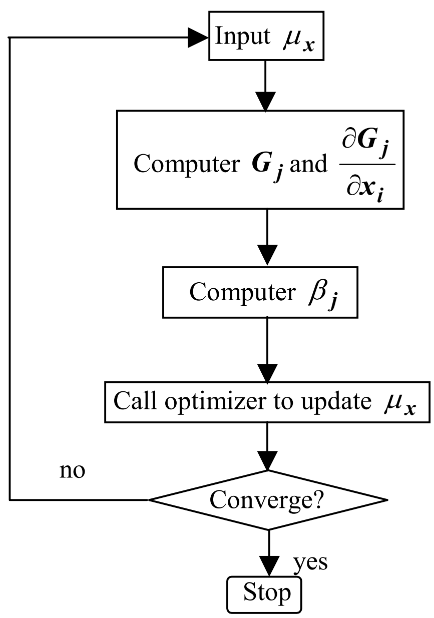

Fig. 3 Flowchart of MVFORM Method

Fig. 3 gives the flow chart of the MVFORM. The MVFORM is very simple and it works just like the deterministic optimization problem. It saves lot of computational cost since all the functions and their derivatives include objective function and constraints are calculated at the mean value μ_x. However, the MVFORM approach has some deficiencies that were observed by many researchers. The function $g(.)$ is liberalized at the mean values of the x_i variables. When $g(.)$ is nonlinear, significant error may be introduced by neglecting higher order terms. A hybrid procedure that combined both MVFORM and SLSV method can be used in practical problem. In the example section, we will show how this hybrid procedure works.

5 Application To Vehicle Crash Safety

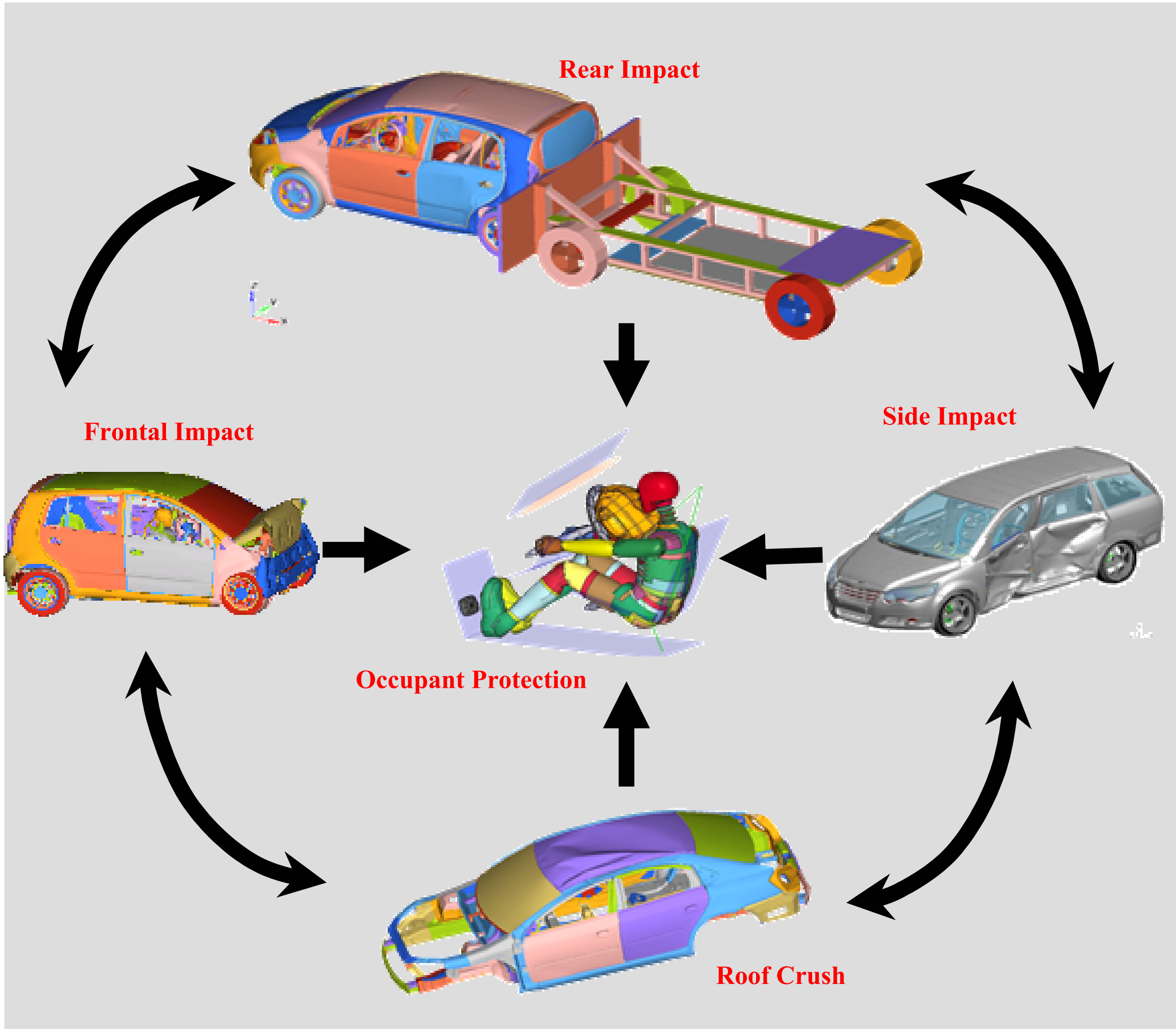

Fig. 4 Multi disciplinary robust design example of vehicle crashworthiness

The methodologies discussed above have been applied to the vehicle structural weight reduction for crashworthiness, noise and vibration. The design problems for vehicle crash safety are particularly complex, in terms of understanding the problem and defining design requirements and design alternatives. They also involve a very high degree of uncertainty and variability. A balance must be struck in designing for crashworthiness between designing the vehicle structure to absorb/manage the crash energy (through structure deformation) and maintaining passenger compartment integrity, all in the face of uncertainty.

With the expense of physical crash testing and with recent increases in computing power and improvement in numerical modeling techniques, much of crash analysis is done today using computer simulations models. However, these simulations bring a different kind of expense. Vehicle finite element models for crash simulation can be upwards of more than 250,000 elements. The required run time can range from 24 hrs to over 100 hours for a single analysis. Even with parallel processing, the expense of crash simulations prohibits extensive optimization, let alone probabilistic optimization. In addition, solver instability often prevents successful application of sequential (optimization) methods, since runs may fail due to design perturbation. It is essential to create approximate models from a number of actual simulations.

The methods discussed above can also be applied to multi-disciplinary robust design problems as shown in Figure 4. For reasons of confidentiality, detailed results are not given in this paper.

6 Summary

A large-scale reliability-based design optimization methods such as robust design for vehicle crashworthiness has been presented in the paper. Two nonlinear response surface methodologies, sequential replacement second order polynomal and moving least square method, along with uniform Latin hypercube sampling method have been investigated shown to be effective.

The robust design has been formulated as a reliability-based design optimization problem. The deterministic optimization, reliability-based design optimization and robust design methodologies have been dicussed and demonstrated.

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